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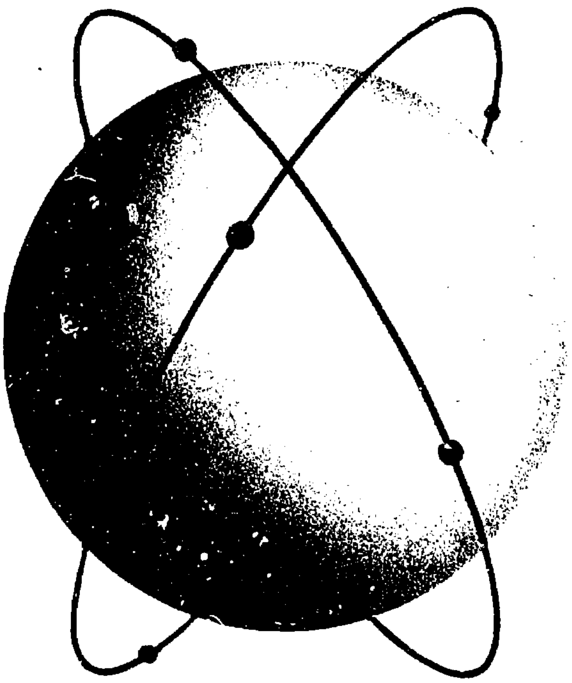
ABSTRACT

As the tenth lesson of the Articulated Multimedia Physics Course, instructional materials relating to circular motion are presented in this study guide. The topics are concerned with instantaneous velocity, centripetal force, centrifugal force, and satellite paths. The content is arranged in scrambled form, and the use of matrix transparencies is required for students to control learning activities. Students are asked to use magnetic tape playback, instructional tapes, and single concept forms at the appropriate place in conjunction with the worksheet. Included are a homework problem set and illustrations for explanation purposes. Related documents are SE 015 963 through SE 015 977. (CC)

ARTICULATED MULTIMEDIA PHYSICS



LESSON 10

NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

IMPORTANT: Your attention is again called to the fact that this is not an ordinary book. It's pages are scrambled in such a way that it cannot be read or studied by turning the pages in the ordinary sequence. To serve properly as the guiding element in the Articulated Multimedia Physics Course, this Study Guide must be used in conjunction with a Program Control equipped with the appropriate matrix transparency for this Lesson. In addition, every Lesson requires the availability of a magnetic tape playback and the appropriate cartridge of instructional tape to be used, as signaled by the Study Guide, in conjunction with the Worksheets that appear in the blue appendix section at the end of the book. Many of the lesson Study Guides also call for viewing a single concept film at an indicated place in the work. These films are individually viewed by the student using a special projector and screen; arrangements are made and instructions are given for synchronizing the tape playback and the film in each case.

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NEW YORK INSTITUTE OF TECHNOLOGY

Old Westbury, Long Island

New York, N.Y.

ARTICULATED MULTIMEDIA PHYSICS

Lesson Number 10

CIRCULAR MOTION

Let your imagination carry you back through more than 2,000 years of man's history, back to Plato in the Greece of the fourth century B.C. This great philosopher was a teacher whome we can imagine speaking to his pupils in a classroom not too different from yours. We must paraphrase his lecture because there is no documentation for his exact words: "The stars," he begins, "eternal, divine, and unchanging lights in the heavens, move around the Earth once each day, as we can see, in that eminently perfect path, the circle. The planets, however, seem to follow erratic paths; they wander through the sky as the year progresses. Yet they, too, are heavenly and divine; they, too, must follow the perfect path of heaven, the circle. And so I set forth to you a problem: determine what uniform and ordered circular motions must be assumed for each of the planets to account for its apparent irregular wanderings."

We give you Plato in our introduction to this lesson because he and his successors in later times and other places exemplify man's early preoccupation with circular motion in his endeavor to explain celestial events without violating either of his two basic faiths: his belief in the divinity and perfection of the circle and his conviction that the chaotic, tumbling motion of the heavenly spheres could be reduced to simple, ordered, logical systems.

Please go on to page 2.

The seeds of many of our modern scientific methods lay in the fertile thoughts of the ancient Greeks. Despite superstition, the study of circular motion in these ancient times as applied to a sky which seemed so near has slowly led to our understanding of the present-day universe:

1. The stars do not move about the Earth in circles.
2. The planets do not move about the Earth or the sun in circles.
3. The orbit of the moon is not a circle.

In short, there is nothing divine or eternal about the circle as Plato thought. The circle is a special kind of geometric figure with special properties; but the same may be said of the ellipse, the square, or the triangle. Nevertheless, planetary and satellite orbits are smooth, closed curves which often closely approximate circles. This lesson will cover motion in curved paths.

As it happens, circular motion is more readily analyzed than elliptical, or hyperbolic, or parabolic motion. Many familiar objects move in perfect circles: the wheels of your car, the edge of the record on your phono playback, the tub in your washing machine, the blades on your electric fan.

We ask, then, what makes circular motion different from motion along a straight line? Does velocity have the same meaning for both motions? How does acceleration differ in circular motion? What forces are present in rotating systems? These and other related questions will be answered in this lesson.

Please go on to page 3.

Somewhere in your reading or in grade school you have met the notion of centrifugal force. You whirl a stone around in a circle at the end of a string in apparent defiance of gravity. What keeps the string taut? Why doesn't the stone fall when it reaches the top of its vertical circular path? "Why, it's obvious," you say. "When a body is whirled around, there is an outward force acting on it. In this case, the force is large enough to counteract the downward pull of gravity. That's why we call this force centrifugal force; the word means 'flying away from the center.' That's exactly what the stone would do if you cut the string. It would fly outward immediately due to centrifugal force."

Consider another manifestation of centrifugal force. When the car in which you are a passenger suddenly rounds a sharp turn, you often find yourself sliding along the seat toward the outside of the curve. You were stationary with respect to the car before you began to slide; hence your body is accelerated. Being well-versed in Newton's laws of motion, you know that a force must have acted on your body to cause the outward acceleration; you can even write a mathematical equation ($F = ma$) to prove the existence of this outward-going force. You conclude that the same force that acted on the string--centrifugal force--is also responsible for your sliding along the seat of the automobile.

Please go on to page 4.

The concept of centrifugal force is quite simple and clear-cut, isn't it? It explains so many things in a forthright, uncomplicated manner. However, explanations of effects in rotating systems based upon centrifugal force have one defect: THEY ARE INCORRECT! THERE IS NO SUCH FORCE AS CENTRIFUGAL FORCE ACTING ON THE STONE ON THE STRING OR ON YOUR BODY IN THE CAR! For more years than we care to mention, textbooks and teachers have been either implying or actually dispensing false information on the subject.

So, when a belief like this, cherished over the years, is shattered, we begin to view other commonly accepted ideas with suspicion. This is exactly the attitude we should like to see develop in you. Very often in physics, "self-evident truths" turn out to be the wildest kind of untruths! The answer is, clearly, that there is no such thing as a self-evident truth. In science, a so-called fact of today may very well turn out to be the fairy tale of tomorrow.

Centrifugal force as an explanation for the behavior of bodies in circular motion is just such a fairy tale. If abandoning the simplicity of the centrifugal force concept disturbs you, don't fret. The true explanation of the taut string of the whirling stone, of the slide along the automobile seat, and of the tearing tension you feel in your arms when you are the last man on an ice-skating "whip" is so much more logical and straight-forward, that once you have grasped it, you will never again regret the loss of "centrifugal" force.

Good luck in the work ahead. We know you will find it interesting.

Please turn to page 154 in the blue appendix.

There are places in deep space which are so remote from the nearest star or planet that the effects of gravitation in these areas are negligible. We can imagine a block of meteoric matter moving through such a space, having been cast of an exploding star thousands of years ago. No forces of any kind act upon it. It is a completely free agent unaffected by gravitation, electrical forces, or magnetic forces.

In the absence of force, the meteor is in dynamic equilibrium. Its motion is accurately described by Newton's First Law.

Just what kind of motion would the meteor have?

(1)

A Uniformly accelerated motion.

B Uniform speed; variable in direction.

C Uniform velocity.

YOUR ANSWER --- B

Refer to Figure 15 on page 46. You can't say that DC is the radius of the circle of rotation shown in the figure. This line segment is too long; Point D is outside the circle. If you used DC as a radius to draw a circle around C, you would find this circle much larger than the one in the figure.

Please return to page 46 and choose a better answer.

YOUR ANSWER --- C

One of the conversions is incorrect. Perhaps it will help if we remind you that there are 100 cm in one meter and 1,000 g in one kilogram.

When you find this single error, the list will then be correct. Locate the error; then return to page 56 and choose the right list of values.

YOUR ANSWER --- C

You are correct. At the instant of removal of the centripetal force, the particle at once is restored to a state of dynamic equilibrium. Since at this instant the motion is tangent to the circle, the particle continues to move along the line of $\vec{v_2}$.

Hereafter in this lesson, we will use the symbol $\vec{v}$ to refer to the "instantaneous velocity" of the rotating particle. Since the direction of motion will be constantly changing, we will refer to "speed" of the particle, the instantaneous velocity of which is $\vec{v}$.

Now let's imagine that the centripetal force is applied to the particle with constant magnitude, continuously changing its direction so that its vector constantly points toward the center. The path of the particle will then be a perfect ________.

What's the missing word? Write it; then check your thinking by turning to page 9.

CORRECT ANSWER: If the magnitude of the centripetal force is constant and always directed toward the center, the path of the particle will then be a perfect circle.

In Figure 9, the symbol F_c is used for centripetal force at every point in the path of the particle and v is used for each instantaneous velocity because the speed of the particle is constant despite the changing direction. The radius of this circle is symbolized by r and we'll call the mass of the particle m.

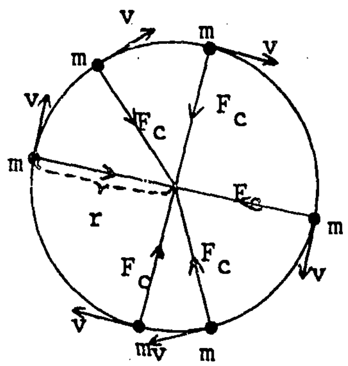


Figure 9

Next we turn our attention to the magnitude of the centripetal force, F_c. Can the required centripetal force F be determined for any given set of conditions where the mass m, the instantaneous velocity v, and the radius of the circle r are determinable? By "required" we mean: can we find the magnitude of the centripetal force F_c needed to keep a particle of mass m moving at a speed v in a circle of radius r? The answer is yes. An equation can be derived which relates F_c to m, v, and r. The derivation is difficult, however, and beyond the level of our course. Rather than burden you with the formal derivation, we're going to obtain the equation with the assistance of our knowledge of units and unit checks.

NOTEBOOK ENTRY
Lesson 10

2. Centripetal Force

(a) A particle moving in a circle with constant speed is acted upon by a force of constant magnitude acting toward the center of the circle. This is centripetal force. (Copy Figure 9 into your notebook.)

Please turn to page 155 in the blue appendix.

To obtain the relationship between F_c on the one hand and m, v, and r on the other, we'll investigate the effect of each of the latter on the force needed to maintain circular motion; but we'll handle them separately. To do this, we'll try to make use of your personal observations and experiences.

We shall begin by studying the relationship between F_c and v. Imagine that you are holding a on d length of string between your fingers, having first wound a few turns c around your forefinger to prevent inadvertent slip. Attached to the string is a "particle," (perhaps a tennis ball). You whirl the particle in a horizontal circle at a relatively slow speed above your head and note the amount of centripetal force (F_c) you have to exert to keep it in this circle. Now assume that you increase the speed of whirling to twice or three times its former value and again note the magnitude of the required centripetal force. Select one of the statements below which fits your observations of what occurs.

(8)

A As the speed increases, the required centripetal force remains the same.

B As the speed increases, the required centripetal force decreases.

C As the speed increases, the required centripetal force increases.

YOUR ANSWER --- B

Your algebra is faulty. Remember, dividing a fraction like mv^2/r by a whole number such as m is the equivalent of multiplying the fraction by the reciprocal of the denominator.

$$a_c = \frac{mv^2}{r} \times \frac{1}{m}$$

So you can see that the answer you chose is not correct.

Please return to page 27 and select the alternative answer.

YOUR ANSWER --- D

There is actually nothing wrong with this answer from a strictly mathematical point of view. The error, if one may call it that, lies in the basic presentation of the result.

It is conventional to isolate the constant k from the variables by showing it as a separate quantity, the first term on the right side. Instead of writing $F_c = m\frac{kv}{r}$, it is preferable to indicate the same relationship this way:

$$F_c = k\frac{mv}{r}$$

In the last analysis, both relationships are mathematically identical, but the latter contains better physics.

Please return to page 59 and choose the best answer.

YOUR ANSWER --- A

Refer to Figure 18 on page 75.

Path CD is a perfectly straight line. The only conditions that will permit any object moving in space to follow such a path are those in which the net force acting on the object is zero. The fact that a large amount of driving force is applied horizontally to the satellite does not negate the gravitational attraction of the Earth. Regardless of the speed of projection of the satellite at point C, then, the net force acting on it is not zero because it is still being attracted by the Earth. Hence, it cannot move along a straight line because it is not in dynamic equilibrium.

There is a much better answer to the question than the one you chose. Return to page 75 please.

YOUR ANSWER --- C

Almost, but not quite!

You worked the movement of terms correctly, but you should have ended with the following equation:

$$v^2 = \frac{F_c r}{m}$$

What happened to the square sign on the v? You appear to have dropped it. Why?

To find v, you must take the square root of both sides of the equation shown above.

Please return to page 81 and then choose the right answer.

YOUR ANSWER --- C

All right, let's go over the essential points.

We started with a series of rough experiments. We whirled a particle on a string and, one at a time, changed each of the three variables: speed, mass, and radius of rotation. In each part of the experiment, we tried to determine what happened to the required centripetal force, F_c, as each of the variables was altered.

We found that the centripetal force had to be increased:

(1) when the speed (v) was increased. So we assumed that F_c was directly proportional to v; that is, $F_c = kv$.

(2) when the mass (m) was increased. Again we guessed that there might be a direct proportion between F_c and m; that is, $F_c = km$.

(3) when the radius of rotation (r) was decreased. This suggested that F_c might be inversely proportional to r, and we proceeded to make this assumption; that is, $F_c = k/r$.

These three proportions were then properly combined into the form:

$$F_c = k\frac{mv}{r}$$

We decided to do a unit check on the relation as it stood. But before we could do this, we had to make another assumption. We assumed which of the following?

(13)

A We should like the force to come out in newtons.

B <u>k</u> equals 1 and has no units.

YOUR ANSWER --- B

You are correct. Centripetal force, regardless of its source, acts along the radius of the circle of rotation toward the center. This radius is always perpendicular to the tangent through the particle at every point of the circle in which the particle is moving.

Try this problem. An electron has a mass of 9.1×10^{-31} kg. Under the action of a magnetic force, an electron moves in a circle of 2.0 cm radius at a speed of 3.0×10^{6} m/sec. At what speed will a proton (mass = 1.6×10^{-27} kg) move in a circle of the same radius if it is acted upon by the same force?

Let us first write the equation for the centripetal force acting on the electron. Thus:

$$F_e = \frac{m_e v_e^2}{r_e}$$

(Copy this.)

The subscript "e" relates all of these quantities specifically to the electron.

Using the subscript "p" for quantities relating to the proton, write the centripetal force equation for this particle.

Please turn to page 17 to check the equation.

CORRECT EQUATION: $$F_p = \frac{m_p v_p^2}{r_p}$$

Now the problem states that the same force is acting on both particles. This tells you at once that $F_e = F_p$ which permits you to equate the right side of one equation to the right side of the other so that:

$$\frac{m_p v_p^2}{r_p} = \frac{m_e v_e^2}{r_e}$$

One of the conditions of the problem is that the proton is to move in a circle of the same radius as the electron. Therefore, you also know that $r_p = r_e$. If these are set equal to each other, the equation reduces further to which of the following?

(22)

A $m_p v_p r_p = m_e v_e r_e$

B $m_p v_p^2 = m_e v_e^2$

C $m_p v_p r_e = m_e v_e r_p$

D None of the equations given above is correct.

YOUR ANSWER --- D

Refer to Figure 3 on page 115. Your answer is improperly limited. To determine where no unbalanced force acts on a particle moving with uniform speed, you must locate those ranges where the particle moves in a perfectly straight line. True, A to C is a straight line, but so is the range from F to G. Over both of these ranges, the particle is in dynamic equilibrium since neither the speed nor direction of the motion is changing. Hence, there is no unbalanced force acting on the particle over either of these ranges.

Please return to page 115. Then select an alternative answer.

YOUR ANSWER --- C

This answer is not right.

Think back to the way we obtained the equation for centripetal force. In deriving the expression for centripetal force, m always represents a single mass, not the combined mass of two different bodies, regardless of their relationship to each other. Remember also that the Earth provides gravitational attraction.

Please return to page 86. You can find the right answer.

YOUR ANSWER --- B

This is correct. We will want v all alone on the left side of the expression. So we have:

$$\frac{m_s v^2}{r} = G\frac{m_s m_e}{r^2}$$

To solve for v, we like to attack the simplification this way: m_s and r on the left can be eliminated by multiplying this side of the equation by r/m_s. But if you multiply one side of any equation by a certain factor, you must do the same thing to the other side. Thus,

$$\frac{r}{m_s} \times \frac{m_s v^2}{r} = G\frac{m_s m_e}{r^2} \times \frac{r}{m_s}$$

We want you to simplify this right down to the point where you obtain v (not v^2) all alone on the left side. Go to it! When you have finished (it's really not difficult at all), compare your answer with those below and select the statement that you feel is correct.

(33)

A $v = G\frac{m_e^2 m_s}{r^3}$

B $v = G\frac{m_e m_s}{r}$

C The equation does not simplify to either of the two expressions shown above.

YOUR ANSWER --- B

Refer to Figure 18 on page 75.

Path CD is a perfectly straight line. An object moving through space can move along a straight line only if it is in dynamic equilibrium, that is, if the net force acting on it is zero. At an altitude of 1,000 miles, the Earth's gravitational pull may be somewhat weakened, but it is still there and constitutes a very definite unbalanced force that would not permit the satellite to move in a straight line. In short, the Earth's gravitational force even at an altitude of 1,000 miles makes it impossible for the satellite to move in perfect dynamic equilibrium.

Please return to page 75. The correct answer should now be evident.

YOUR ANSWER --- A

Although this statement is true, it is not included in the context of the notebook entry in question. Check your notes again.

Please return to page 133. You'll have to try another answer.

YOUR ANSWER --- B

This answer doesn't follow from our reasoning. If a particle is going to change its direction of motion at all, a force must be applied to it in the direction of the change. When the string is cut, the only existing inward radial force vanishes; hence the particle certainly could not begin to move inward at exactly the time when the only force that could cause it to do so ceases to exist!

You must remember that the particle is moving at a tangent to the circle at every instant in time and that the centripetal force is applied for the purpose of changing the motion from linear to circular motion. So if the centripetal force causes the motion to change from linear to circular, the removal of the force must permit the particle to return to linear motion along the same line it was following at the instant when the string was cut. But was the particle flying inward toward the center along the radius at the instant of cutting? Which way was it going?

You should have no difficulty in choosing the correct answer now. Please return to the question by turning to page 150.

YOUR ANSWER --- A

You are correct. As the radius decreases, the circle of motion becomes smaller and more curved. The increased curvature is a greater deviation from a straight-line path and hence requires more centripetal force to produce it. Refer to Figure 10.

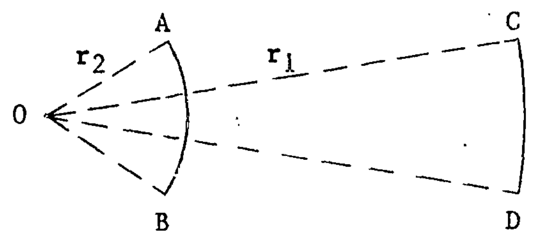


Figure 10

Arc AB is part of a circle with small radius r_2; arc CD is part of a circle with larger radius r_1. Both arcs are about the same length indicating the same distance traveled in unit time. CD is more nearly a straight line than AB; hence it requires less centripetal force to produce it.

Thus as r decreases, F_c increases. Here again, we see a suggestion of a proportion but this time, an inverse one. Following the same procedure as before, we assume that F_c is inversely proportional to r and write:

$$F_c = \frac{k}{r}$$

So far, we have two assumed proportions:

$$F_c = kv \quad \text{and} \quad F_c = \frac{k}{r}$$

That is, centripetal force is directly proportional to particle speed and inversely proportional to the radius of rotation. What is the other factor whose relationship to F_c we wish to determine?

Write it; then turn to page 25, please.

CORRECT ANSWER: The other factor whose relationship to F_c we wish to determine is <u>mass</u> (m).

First, we'll repeat our observations about the other terms.

(1) When the speed of a rotating particle <u>increases</u>, all other factors remaining the same, the centripetal force required to maintain circular motion <u>increases</u>.

(2) When the radius of rotation of the particle <u>increases</u>, all other factors remaining the same, the centripetal force required to maintain circular motion <u>decreases</u>.

Please turn to page 156 in the blue appendix.

Now how about mass? If in the imaginary experiment you had used a heavy object instead of a light one, say, a stone rather than a tennis ball, would you have had to exert more or less centripetal force as compared to that of the previous case for the same radius and the same speed?

(10)

A More force.

B Less force.

YOUR ANSWER --- A

You are correct.

We know, too, that the motion of a particle in a circular path may be described in terms of the following expression:

$$F_c = \frac{mv^2}{r}$$

The force in this case (centripetal force) produces an acceleration directed toward the center; hence this force may be substituted for the general value of F in a = F/m, where a is the centripetal acceleration and m is the mass of the particle. In other words, we write:

$$a = \frac{F}{m} \quad \text{(general form of Second Law)}$$

But since $F = F_c = \frac{mv^2}{r}$ (centripetal force equation),

we may substitute $\frac{mv^2}{r}$ for F in the general form and write:

$$a_c = \frac{\frac{mv^2}{r}}{m}$$

where a_c stands for centripetal acceleration.

Simplify this last expression. What do you get?

(28)

A $a_c = \frac{v^2}{r}$

B $a_c = \frac{m^2v^2}{r}$

YOUR ANSWER --- A

Not so.

If all four assumptions had been correct, then F_c would have come out in newtons or kg-m/sec^2 instead of coming out in kg/sec as it did.

Let's review the assumptions once again:

(1) We found that F_c increased when m increased, so we assumed a direction proportion and wrote $F_c = km$.

(2) We found that F_c increased when v increased, so we again assumed a direct proportion and wrote $F_c = kv$.

(3) We found that F_c increased when r decreased, so we assumed an inverse proportion and wrote $F_c = k/r$.

(4) We assumed k equal to unity; we also assumed k to be a pure number without units.

Now, since F_c did not come out in newtons or kg-m/sec^2, then <u>one or more of these assumptions must be incorrect</u>.

Please return to page 66. You know the answer now.

YOUR ANSWER --- C

Not so! This equation shows acceleration to be proportional to the product of force and mass. Do you remember the verbal form of the Second Law? The acceleration of a mass is directly proportional to the applied force, but it is inversely proportional to the mass of the body.

Write the correct form of the Second Law equation; then turn to page 98 again and choose the correct answer.

YOUR ANSWER --- C

Sure, that's correct. As the speed increases, the need to pull the string toward the center of rotation with more force is quite evident. Thus as the speed increases, the centripetal force increases. This suggests the possibility of a direct proportion between F_c and v. Remember, this is an intelligent guess but a guess nevertheless. Actually F_c might be proportional to v^2 or to $\sqrt[3]{v}$ or to v^3, or there may be no true proportionality at all. So by assuming that F_c is directly proportional to v, we are going "out on a limb," but there is no harm in this as long as we are aware of what we are doing. Obviously, such an assumption will have to be tested later on.

Now going along on this assumption, we'll write the assumed direct proportion in our familiar mathematical form:

$$F_c = kv$$

Next, picture repeating the experiment with a string only half as long, this time keeping the speed constant at all times. What happens this time?

(9)

A As the radius decreases, the required centripetal force increases.

B As the radius decreases, the required centripetal force decreases.

YOUR ANSWER --- B

You are quite correct. These are the right MKS values for this problem.

Now you have the equation and the values for substitution. Suppose you solve the problem and let us have the answer to two significant figures.

What is the maximum speed of the ball?

(20)

A 4.2 cm/sec

B 18 m/sec

C Neither of the above answers is correct.

YOUR ANSWER --- A

You're letting some old, fixed ideas get the better of your reasoning. You were probably thinking of one possible way of starting the particle in its circular path (as when particles are affixed to strings)--by having the string slack before rotation begins; then you pictured the particle moving away from the center of rotation before the string became taut.

Bear in mind that the force F (any one of the infinite number of centripetal forces for any one of the infinite number of possible positions the particle may have at a given instant) cannot be applied until the string is taut. Thus, any movement the particle undergoes while there is still slackness in the string must occur before the centripetal force is applied.

Once the string is taut, the particle cannot possibly move farther from the center, and hence it could not accelerate as described in this answer.

Please return to page 79 and choose a better answer.

YOUR ANSWER --- A

Acceleration means a time rate of change of velocity. Acceleration occurs only when an unbalanced force acts on a mass. From the Second Law:

$$a = \frac{F}{m}$$

it is evident that regardless of the nature of the mass, if the unbalanced force is zero, then there is no acceleration. If F = 0, then:

$$a = \frac{0}{m} \text{ so that } a = 0$$

since zero divided by any number is zero.

The meteor is not acted on by an unbalanced force of any kind; this is the condition we have set up. Thus, F = 0 and a = 0, so the meteor cannot be moving with accelerated motion, uniform or otherwise.

Please return to page 5 and choose a better answer.

YOUR ANSWER --- B

Certainly, the equation you derived shows no such relationship. The mass of the satellite (m_s) does not even appear in it.

Perhaps you thought that the greater weight would require greater speed to provide the inertia to keep the satellite from falling to the Earth What you probably forgot was that the weight of the satellite provides an increase in inertia.

Since m_s does not appear in the equation for satellite speed, what can you conclude about the dependence of satellite speed on satellite mass?

Please return to page 52 and make another answer selection.

YOUR ANSWER --- A

No.

You're forgetting that the radius of the orbit is the distance between the satellite and the center of the Earth. Since the radius of the Earth is taken as roughly 4,000 miles, then the altitude must be added to this figure to obtain the orbital radius.

Please return to page 53 and choose the alternative answer.

YOUR ANSWER --- C

You are thinking of acceleration as a change of speed only. Reflect for a moment. The instantaneous velocities (v_1, v_2, and v_3) all have the same magnitude, but the directions are different. Now, although you may think of acceleration as the change of magnitude of a velocity in many cases, there are just as many situations where an applied force produces acceleration by changing the direction of the velocity rather than its magnitude. In the following definition:

$$a = \frac{\Delta v}{\Delta t}$$

The delta preceding the v does not specify whether the magnitude or the direction (or both) is changing. Acceleration occurs if either one varies with time, or if both do simultaneously.

So, the unbalanced force F in Figure 12 on page 79, when in any one of its possible positions, must cause the particle to accelerate. The question is, in what direction does the acceleration take place?

Please return to page 79. Try another answer.

YOUR ANSWER --- D

One of the conversions is not right. It might help if we reminded you that there are 100 cm in one meter and 1,000 g in one kilogram.

When you locate the error, the list of values will be correct. Find it; then return to page 56 and select the right list.

YOUR ANSWER --- B

This is incorrect. It looks as if you were confused by a decimal point. Or you might have doubled the speed rather than squaring it.

The equation is:

$$F_c = \frac{mv^2}{r}$$

Now substitute the numbers only:

$$F_c = \frac{80 \times (20)^2}{20}$$

If you doubled the 20 in the numerator instead of squaring it, you would obtain:

$$F_c = \frac{80 \times 40}{20}$$

$$F_c = 160 \text{ units of force}$$

Of course, this is wrong. Repeat the calculation; then return to page 104 and select a better answer.

YOUR ANSWER --- A

Refer to Figure 3 on page 115. The path of the particle from A to C is a straight line, and since you are told that you are to assume uniform speed throughout the path, then there is no unbalanced force acting on the particle from A to C. You answer is correct thus far.

From D to E, the particle's path is a smooth curve. As we have seen, a particle will traverse a curved path only if an unbalanced force acts on it while it is following the curve. Hence, there <u>must be</u> an unbalanced force acting on the particle throughout the time that it is moving from D to E. Thus this part of your answer is incorrect.

Please return to page 115 and choose a better answer.

YOUR ANSWER --- A

You are correct. Since m for Mars is 1/10 m for Earth, then v for the Martian satellite would be smaller than v for the Earth satellite, and the Martian satellite would travel more slowly.

How much more slowly would it travel? That is, at about <u>what fraction</u> of the Earth satellite's speed would the Martian satellite move?

(39)

A $\frac{1}{100}$

B $\frac{1}{10}$

C $\frac{1}{3}$

YOUR ANSWER --- B

You are correct. In Trials 1 - 3 only the mass varies; in Trials 4 - 6 only the speed varies; in Trials 7 - 9 only the radius varies. Now let's find out what we can learn from these data.

Suppose we consider Trials 1 - 3 first. For these three trials, v = 1.0 m/sec and r = 1.0 m throughout, while the mass goes from 1.0 kg to 2.0 kg to 3.0 kg. If we take 1.0 kg as the initial mass, the centripetal force required is 1.0 nt initially. Now the mass is <u>doubled</u> to 2.0 kg, and we find that the force has also <u>doubled</u> to 2.0 nt. Last, the mass is <u>tripled</u> from 1.0 kg to 3.0 kg, resulting in a <u>tripling</u> of the force from 1.0 nt to 3.0 nt.

Based upon such functional manipulations, the answer to this question should be immediately forthcoming: is F_c directly proportional to m? Does F_c vary directly as the mass of the rotating body? What do you say? Yes or no?

Write your answer; then turn to page 42.

CORRECT ANSWER: Yes. The centripetal force F_c is directly proportional to the mass m of the body in circular motion.

All right, then, this was the assumption we made, and it has been verified by experiment. We now know definitely that $F_c = km$.

Turning our attention next to Trials 4 through 6, we observe that the speed has been made to vary while the mass and radius were held constant. Let's summarize part of the chart information below:

(m and r constant)	Speed	Centripetal Force
(Trial 1)	1.0 m/sec	1.0 nt
(Trial 4)	2.0 m/sec	4.0 nt
(Trial 5)	3.0 m/sec	9.0 nt
(Trial 6)	4.0 m/sec	16.0 nt

It is obvious that doubling the speed does not double the centripetal force, tripling the speed does not triple the force; nor does quadrupling the speed quadruple the force. Therefore, F_c is not directly proportional to v. A proportionality does exist, however, which can be recognized from the above data. Which one of the following expresses this correctly?

(15)

A $F_c = kv^2$

B $F_c = kv^3$

C $F_c = k\sqrt{v}$

YOUR ANSWER --- C

Refer to Figure 3 on page 115. You are quite correct. Over both of these ranges, the speed and direction of the particle are unchanging; thus the particle is in dynamic equilibrium and is not acted upon by any unbalanced force.

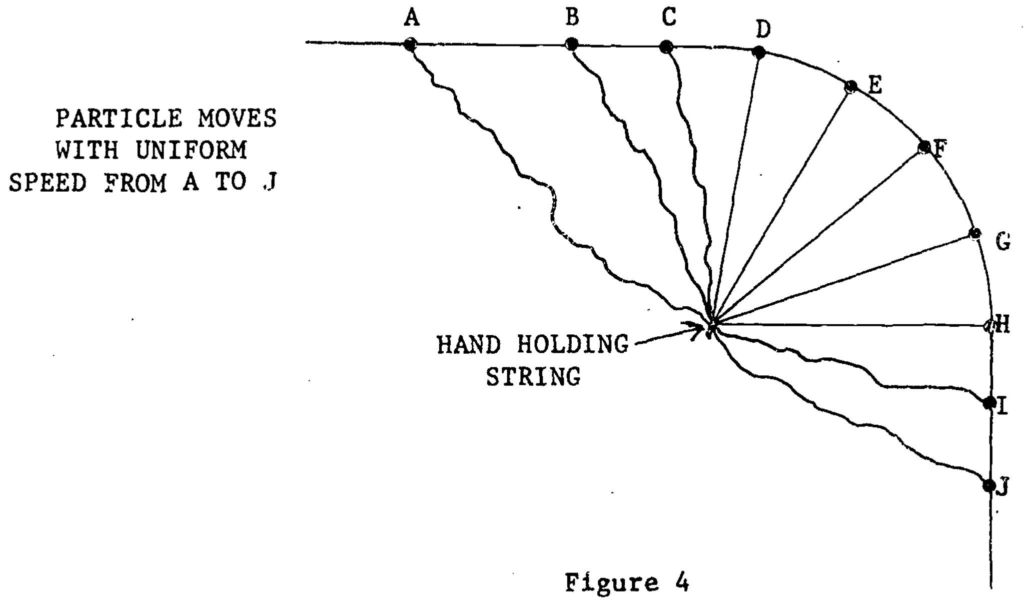


Figure 4

Now refer to Figure 4. A particle is shown in various parts of a path from A to J, moving with uniform speed at all times. A boy holds a string in his hand at point O so that when he wishes, he can take up the slack of the cord and exert a force on the particle (a ball, perhaps). Note that the string is shown in its slack condition when the particle is at points A, B, C, I, and J. At the remaining points in the path (D, E, F, G, and H), the boy has pulled the string taut and is exerting an unbalanced force on the particle. Assuming that this is the result of an experiment using sequential high speed flash photography, how do you know that the string is not exerting an unbalanced force on the particle at A, B, C, I, and J?

Think about this; then turn to page 44.

Two separate aspects of the diagram (Figure 4 on page 43) indicate that the string does not apply an unbalanced force to the particle at A, B, C, I, and J.

First, the string is slack. A slack string cannot exert a force on anything until it is pulled taut. Second, at the points named, the particle is moving with constant velocity; hence it cannot be experiencing an unbalanced force in any direction.

Just before the particle reaches D, its path starts to curve. It then continues to curve until it arrives at H. Throughout this interval, the boy exerts a force on the string (note that it is taut) as indicated by the arrows. He keeps the string at a constant length all during the time that the particle moves from D to H, pulling inward toward the finger around which the string is wound. (Assume also that the finger does not move during this time.)

Using the above experimental facts, tell us this: The curve from D to H is part of what special geometric figure?

(4)

A Curve DH is an arc of a circle.

B Curve DH is part of an ellipse.

C Curve DH is part of a parabola.

D I don't know how to determine this.

YOUR ANSWER --- A

You are correct. The centripetal acceleration of a particle moving in a circle with constant speed is given by the quotient of the square of the speed divided by the radius of the circle.

NOTEBOOK ENTRY

Lesson 10

(Item 3)

(b) The centripetal acceleration of a particle moving in a circular path is directly proportional to the square of the speed of the body and inversely proportional to the radius of rotation.

$$a_c = \frac{v^2}{r}$$

Throughout this lesson, we have carefully avoided the use of the word "body" in phrasing the laws. Our consistency in using "particle" rather than "body" arises from the difficulty one has in discussing the behavior of bodies of various shapes in circular motion.

Please turn to page 46.

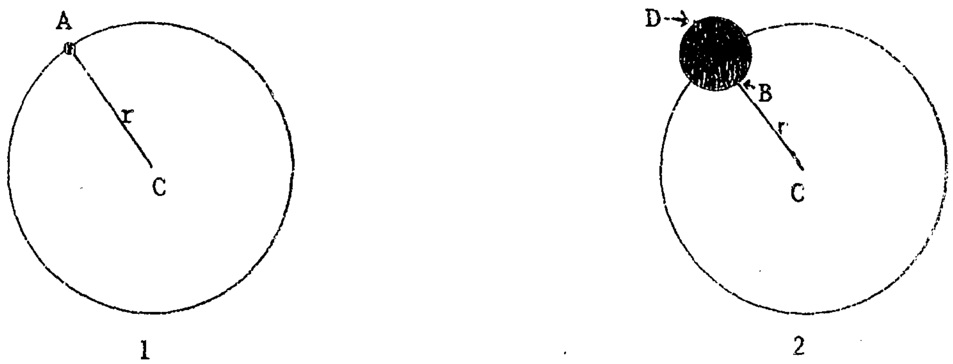


Figure 15

In Figure 15(1) we have a very small sphere. If it were minutely small, we could properly call it a particle, so let us imagine that it is tiny enough to describe as a particle. In that case, the radius of rotation (r) is clearly and unquestionably defined as line segment AC. But in Figure 15(2), we have a <u>body</u>, not a particle. Which line segment is the radius here?

(29)

A BC is the radius of this circle.

B DC is the radius of this circle.

C Neither BC nor DC is the radius of this circle.

YOUR ANSWER --- A

No. A reaction is evidence of Newton's Third Law of Motion at work. In applying the Third Law, you must remember that there are always two bodies involved and that when a force is applied to Body A by Body B, then a force equal in magnitude but opposite in direction is applied by Body B to Body A.

What the answer above really says is that the force applied to the ball (Body A) produces a reaction force also applied to the same body. If this were the case, then centripetal and centrifugal forces would cancel each other and leave the ball in dynamic equilibrium. But if it is in dynamic equilibrium, it must move in a straight line. Hence, by thinking of the two forces as applied to the same body, you would make it impossible for the ball to move in a circle!

Please return to page 64. Choose the alternative answer.

YOUR ANSWER --- C

You made no use of the fact that $r_e = r_p$, the condition given in the statement of the problem. Remember, you were told that the proton was to move in a circle of the same radius as the electron. This fact simplifies the equation considerably.

But a more fundamental error concerns a missing exponent. How did that disappear?

Please return to page 17 and select the right answer.

YOUR ANSWER --- B

You are correct. This is an inevitable conclusion. Acceleration must occur in the direction of the unbalanced force. Regardless of the particular F we choose--and there are an infinite number of possible positions of P and consequently an infinite number of possible directions for F--it is always directed toward the center; therefore <u>the particle must be constantly accelerating toward the center</u>.

Does this give you a feeling that the particle must fall into the center ultimately? Incidentally, this is a very common feeling among people who hear of centrally directed acceleration for the first time. If you wonder about this, we believe we can straighten out your thinking by approaching it from a slightly different viewpoint. Let's try it.

In the situation shown in Figure 12 on page 39, would you say that the centripetal force F (F_1, F_2, F_3, and so on around the circle) is being applied continuously or discontinuously in spurts?

Check your answer by turning to page 50.

CORRECT ANSWER: Centripetal force must be applied continuously if the particle is to move in a smooth circle as this one does.

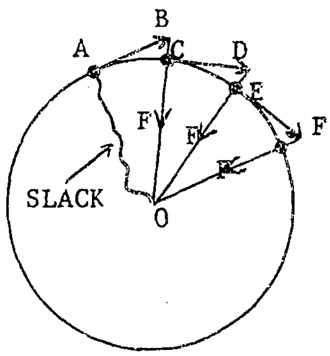


Figure 13

Instead of applying the centripetal force smoothly, what would happen if we applied it discontinuously in a series of jerks toward the center with slack periods in between? In Figure 13, let's imagine that we start with the particle at A, assuming that it is already in motion. The string is slack as shown, so the particle must move in a straight line, say AB. Let AB be the distance covered by the particle while the string is slack; then, suddenly at B the string is tightened and force F applied to it. Say the force acts instantaneously. Thus, instantaneously the particle will be dragged down from B to C, and at the instant it reaches C, the string is allowed to go slack again. At this instant, it will again take off at a tangent to the curve on path CD.

While the string was slack from A to B, the distance between the particle and the center of the circle O was increasing as is evident from the diagram. The particle moved further away from the center by the amount BC. Then upon tightening the string, the particle was brought back to the circle (B to C) by force F, whereupon the string went slack again, and the particle moved from C to D. Along which path was the particle undergoing acceleration toward the center?

(26)

A AB

B BC

C CD

YOUR ANSWER --- A

It may be that you are insensitive to force changes or that you can't recall exactly what happens in this situation.

Think of whirling a ball at a slow speed and then at a much faster speed. Try to picture the inward force you have to exert to prevent the ball from flying off at a tangent. You'll find that there is a definite difference between the required forces for the slow and fast rotational speeds.

Please return to page 10. Choose a better answer.

YOUR ANSWER --- C

You're right! The correct procedure follows:

$$\frac{r}{m_s} \times \frac{m_s v^2}{r} = G\frac{m_s m_e}{r^2} \times \frac{r}{m_s}$$

This leaves you with:

$$v^2 = G\frac{m_e}{r}$$

Then taking the square root of both sides, you get:

$$v = \sqrt{G\frac{m_e}{r}}$$

Examining the terms under the radical, we see that G is known. (Do you remember its value?) It's 0.667 x 10^{-10} m^3/kg-sec^2. Also, m_e is known (5.98 x 10^{24} kg), and r, the radius of the circle of rotation (the orbit of the satellite), can be determined.

But prior to working out satellite velocity problems, we can now answer the second question proposed earlier: Does a heavier satellite have to move faster or more slowly than a light one to stay in the same stable orbit?

Study the final velocity equation above. Then select one of the answers below.

(34)

A A heavy satellite must move more slowly than a light one to follow the same orbit.

B A heavy satellite must move faster than a light one to follow the same orbit.

C Heavy and light satellites move at the same speed when in the same orbit.

D The equation does not answer the question.

YOUR ANSWER --- C

You are correct. Considering G and m_e as constants, to determine the effect of radius on speed, we can rewrite the equation thus:

$$v = \frac{k'}{\sqrt{r}}$$

We see at once that the speed is inversely proportional to the square root of the radius.

The importance of this relationship is nicely shown by the following problem. An artificial satellite must be accelerated in orbit to nearly 18,000 miles per hour at an altitude of 250 miles above the Earth's surface. At what altitude would it have a stable orbit if it were accelerated to only 9,000 miles per hour?

The answer is that the satellite would have to lifted to an altitude of 13,000 miles above the Earth to find a stable orbit at 9,000 mi/hr. Does this sound like a tremendous jump? It does, but there is good reason for it. Let's see how it works out.

What is the radius of the orbit at an altitude of 250 miles?

(36)

A 250 miles.

B 4,250 miles.

YOUR ANSWER --- C

This is not true.

If F_c = k /v, then the chart for these trials would look like this (taking k = 1):

	v	$k\sqrt{v} = F_c$
(Trial 1)	1.0 m/sec	$k\sqrt{1}$ = 1.0 nt
(Trial 4)	2.0 m/sec	$k\sqrt{2}$ = 1.41 nt
(Trial 5)	3.0 m/sec	$k\sqrt{3}$ = 1.73 nt
(Trial 6)	4.0 m/sec	$k\sqrt{4}$ = 2.0 nt

(Taking k = 1 is not actually necessary, but it does simplify the arithmetic.)

Our chart does not show these experimental results for the trials listed. This means that the expression $F_c = k\sqrt{v}$ <u>does not meet the requirements of the data</u> and, therefore, cannot be the correct equation.

Please return to page 42 and make another selection.

YOUR ANSWER --- B

No! You're being careless. If you should forget an equation that states a principle, you can help yourself by thinking of the verbal form of the principle and then making the equation fit the verbal form.

The acceleration of a mass is directly propotional to the force applied and inversely proportional to the mass of the body. Is that what the equation you selected says? Of course not.

Please return to page 98 and select the right answer.

YOUR ANSWER --- A

You are correct. Since you obtained the right literal solution, you apparently don't need further help on this.

In the event that you did not copy the problem specifications, we'll restate the problem: a 980-g ball is whirled in a horizontal circle the radius of which is 36 cm. What is the maximum speed it can have if it is not to break the string? This particular string will break if 49 nt of force are exerted on it.

Only one of the following lists is completely correct for solving this problem. Which is the right one?

(19)

A
$m = 0.98$ kg
$r = 0.036$ m
$F_c = 49$ nt
$v = ?$

B
$m = 0.98$ kg
$r = 0.36$ m
$F_c = 49$ nt
$v = ?$

C
$m = 9.8$ kg
$r = 0.36$ m
$F_c = 49$ nt
$v = ?$

D
$m = 980$ g
$r = 0.36$ m
$F_c = 49$ nt
$v = ?$

YOUR ANSWER --- B

If the direction of a moving body changes, this constitutes acceleration. Do you remember the reasoning that lies behind this statement? Perhaps a very brief review is called for.

Velocity is a vector quantity. The magnitude of the velocity vector is the speed, but the direction of motion must be stated to fully describe a given velocity. If either the speed or the direction of a motion is altered, the velocity has been changed. This is the same as saying that any body which moves in a path having a variable direction must be accelerating while the direction is changing. So, when you say that the path is variable, you are saying that the meteor moves with accelerated motion.

But acceleration occurs only when an unbalanced force acts on a mass. From the Second Law:

$$a = \frac{F}{m}$$

it is clear that, regardless of the nature of the mass, the acceleration is zero if the unbalanced force is zero. That is, if $F = 0$, then:

$$a = \frac{0}{m} \text{ or } a = 0$$

We specified that the meteor is moving in a place where no force of any kind acts on it; hence the unbalanced force is zero. So, its acceleration is also zero, and its path cannot be a varying one.

Please return to page 5 and select a more suitable answer.

YOUR ANSWER --- A

You made no use of the fact that $r_p = r_e$, the condition given in the statement of the problem. Remember, you were told that the proton was to move in a circle of the same radius as the electron.

Furthermore, r_p and r_e are denominators in the original equation; you have moved both of them up into the numerator in an improper mathematical operation.

Please return to page 17; then look at the original equation once again. If $r_p = r_e$, what can you do with these factors immediately?

YOUR ANSWER --- A

You are correct, of course. As the mass increases, the required centripetal force increases. Again assuming a direct proportionality, we write:

$$F_c = km$$

We now have the following three proportions:

$$F_c = kv$$

$$F_c = \frac{k}{r}$$

$$F_c = km$$

As F_c is the dependent variable in all three proportions, we may put them together to form a single statement with F_c on the left of the equals sign. Can you do it? One of the following is the correct combination; the others are wrong. Select the one you think is right.

(11)

A $F_c = k\frac{mr}{v}$

B $F_c = k\frac{r}{mv}$

C $F_c = k\frac{mv}{r}$

D $F_c = m\frac{kv}{r}$

YOUR ANSWER --- A

Refer to Figure 13 on page 50.

The string was slack all throughout the flight of the particle from A to B. Thus there could be no force acting on it in any direction due to the string. With no unbalanced force acting on it, the particle must have moved with uniform velocity from A to B. We must conclude, then, that it did not accelerate at all along this path.

Please return to page 50 and think this over once more before making another answer choice.

YOUR ANSWER --- B

This is not true. Refer to Figure 4 on page 43. You may not know much about the properties of ellipses but you can recognize this: an ellipse cannot be drawn by using a fixed length of string tied to a single fixed center. In other words, we can say that an ellipse has a varying radius when this radius is drawn to various parts of the curve from the same fixed center.

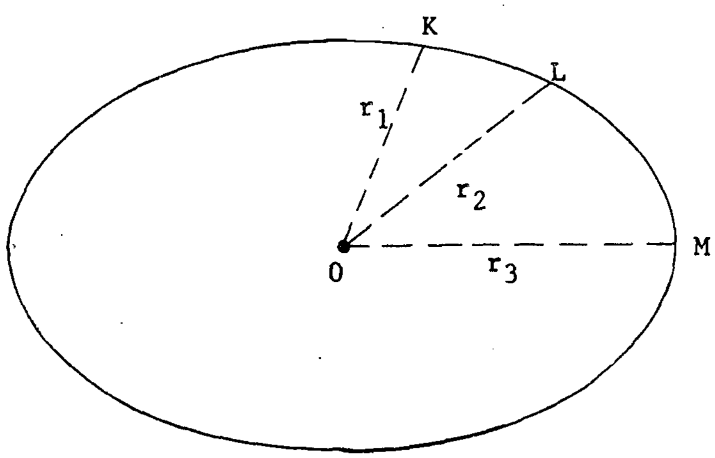


Figure 5

See Figure 5. The three radii, r_1, r_2, and r_3 are of different lengths, so the ellipse could not have been drawn with a fixed length of string tied to point O.

Now think! What kind of geometric figure can be drawn by a fixed radius rotating so that one end of it remains on a single fixed point?

Please return to page 44 and choose the right answer.

YOUR ANSWER --- B

No. The mass of the Earth is implicit in the F_c part of the equation. That is, F_c is provided by the gravitational pull of the Earth on the satellite, and the mass of the Earth is included in the expression for this gravitational pull.

Please return to page 86 and select a better answer.

YOUR ANSWER --- C

You are correct. The speed v is directly proportional to the square root of the planet's mass. Thus $v = k\sqrt{m}$, and, since the mass of Mars is about 1/10 that of Earth, then $v = k\sqrt{1/10}$ or roughly:

$$v = \frac{1}{3}k$$

NOTEBOOK ENTRY

Lesson 10

4. Satellite motion

(a) The speed of an Earth satellite is given by:

$$v = \sqrt{G\frac{m_e}{r}}$$

where v = speed in m/sec, G = constant of universal gravitation in $m^3/kg\text{-}sec^2$, m_e = mass of the Earth in kilograms, and r = radius of orbit in meters.

(b) To find the speed of an orbiting satellite around any planet other than Earth, substitute the mass of this planet for m_e.

(c) For a given speed, the radius of a satellite orbit may be found from:

$$r = \frac{Gm_e}{v^2}$$

Before continuing, please turn to page 158 in the blue appendix.

As we draw near the close of this lesson, you are perhaps wondering if we are going to mention centrifugal force at all. You will have observed of course, that we did not need it at all to explain any of the effects in circular motion. The notion of centrifugal force is quite superfluous in physics; yet we encounter it every now and then in our reading. Let's put the phrase in its proper place here and now.

To eliminate the side effects due to gravity, friction, air resistance, and so on, let us conceive of a ball revolving around a frictionless bearing in a vacuum on the end of a string in deep space where gravitation may be ignored. (Figure 20)

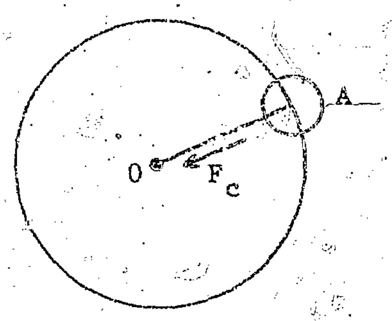


Figure 20

In this idealized situation, there is only one force acting on the ball if it moves with uniform speed. This force is centripetal force, directed toward the center of rotation at every instant. It is the centripetal force which causes centripetal acceleration and prevents the ball from moving off in a straight line.

Now, in the original concept of centrifugal force, it was thought that in some mysterious way a force acted outward from the center on the ball, causing the string to remain taut. We have shown that no such force exists. The only force acting on the ball is the inward one--centripetal force.

In recent years, there has been an inclination among physicists to regard centrifugal force as a reaction to centripetal force and thus change the concept itself. We know that an inward force is applied to the ball; this automatically means that another force having the same magnitude but opposite direction must be applied to which of these?

(40)

A Ball.

B String.

YOUR ANSWER --- C

Statement C is quite accurate but, unfortunately, it has nothing to do with the information in notebook entry 2(h).

Please return to page 133. Try again!

YOUR ANSWER --- C

You are correct. The three proportions should be combined into the single proportion shown in the answer. But remember, please, that the combined form represents a series of three assumptions which have yet to be verified.

As a first step, let us do a unit check on this expression, making still another assumption: we shall assume that k equals 1 and that it is dimensionless. Then, we'll express the mass (m) in kilograms, the velocity (v) in meters per second, and the radius (r) in meters. If all our assumptions are correct, then F_c should come out in newtons.

$$F_c = \frac{kg\text{-}\frac{\not{m}}{sec}}{\not{m}} = \frac{kg}{sec}$$

Were all our assumptions correct?

(12)

A Yes.

B No.

C I don't know.

YOUR ANSWER --- B

There are at least two errors in this solution.

If you work out the units of this expression, you get something like this:

Given: $G = \frac{m^3}{kg\text{-}sec^2}$, $m_e = kg$, $m_s = kg$, $r = m$.

Substituting: $v = (\frac{m^3}{kg\text{-}sec^2})(\frac{kg \cdot kg}{m})$

$$v = \frac{m^2 kg}{sec^2}$$

But velocity is not measured in the square of meters per second times kilograms.

So, work out the problem carefully.

ase return to page 20 and choose another answer.

YOUR ANSWER --- A

You're getting your squares and square roots mixed up again. The speed v is directly proportional to the square root, not to the square, of the planet's mass.

Please return to page 40 and try again.

YOUR ANSWER --- B

If quantity a varies inversely as the square of b, then you would see this form:

$$a = \frac{k}{b^2}$$

However, in the satellite speed equation, the form is entirely different. We might rewrite it as follows:

$$v = \sqrt{G}\,\frac{\sqrt{m_e}}{\sqrt{r}}$$

Since G is the constant of universal gravitation, and the mass of the Earth m_e is constant as far as we are concerned, then the product Gm_e may be replaced by a constant to study the proportion.

$$v = \frac{k}{\sqrt{r}}$$

Surely this form is not the same as $a = k/b^2$, is it? Hence, the speed cannot be inversely proportional to the square of the radius.

Please return to page 122 and choose another answer.

YOUR ANSWER --- A

If the magnetic force is to serve as the centripetal force, then it cannot act along the tangent. A force acting along the tangent could increase or decrease the instantaneous speed of a particle, but it could not change its direction. Remember that the direction of a particle moving in a circle is constantly changing.

Please return to page 124. The other answer is correct.

YOUR ANSWER --- B

This answer indicates that you may not have understood the question or that you can't recall exactly what happens in such a situation. Your finger, through the medium of the string, must exert a centripetal force inward along the radius of the circle as the ball is whirled around. As we have explained, the inward or centripetal force must act toward the center of the circle at all times, requiring you to shift your pulling direction continuously. Normally, the human muscular coordination is more than adequate to make this continuous shift more or less automatic; you don't have to think about it.

Now as the ball is whirled at a greater speed, the need for centripetal force still exists, of course. The question is, will the force required to keep the ball in a circular path be greater, smaller, or the same for high speed as compared with the force at low speed?

Please return to page 10. Select a better answer.

YOUR ANSWER --- C

You are correct. BC is too short and DC is too long for either to be the radius. But this is a real situation; certainly, a ball can be whirled on a string with a definite and fixed radius of rotation.

There is a point in (or near) any body regardless of its shape at which we may consider all of its mass to be concentrated. The mass is not actually concentrated there, of course, but by establishing such an imaginary point, problems in rotation become soluble. For example, if we consider all the mass of a uniform sphere or of a perfect uniform cube to be concentrated at the geometric center, this point may then be taken as the particle which terminates the radius. The point where all of the mass of any body may be considered to be concentrated is called the center of gravity of the body. Our course of study does not permit an extended discussion of the center of gravity of oddly shaped bodies such as cones, pyramids, cylinders, and so forth, but we feel that you should be exposed to the idea very briefly.

One important characteristic of the center of gravity is this: if you place your finger immediately below the center of gravity of an object (along a vertical line), the body will balance on your finger. To see Figure 16 which illustrates this, turn to page 73.

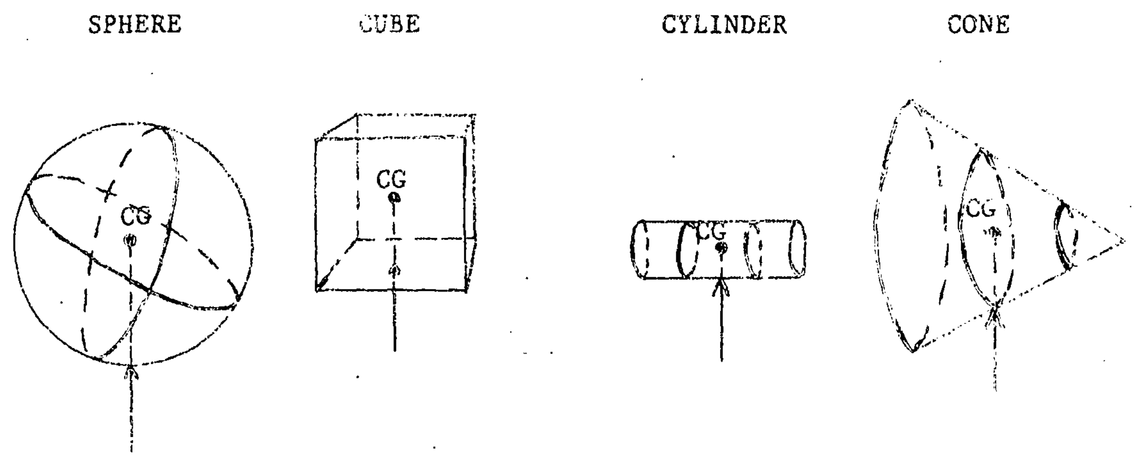


Figure 16

Figure 16 shows 4 geometric objects: a sphere, a cube, a cylinder, and a cone. In each case, the object is balanced on the point of an arrow placed immediately below its center of gravity (CG).

In many solid shapes, the center of gravity coincides with the geometric center. This is true, for example, in a uniform sphere, cube, or cylinder; but it is not true in a cone or pyramid.

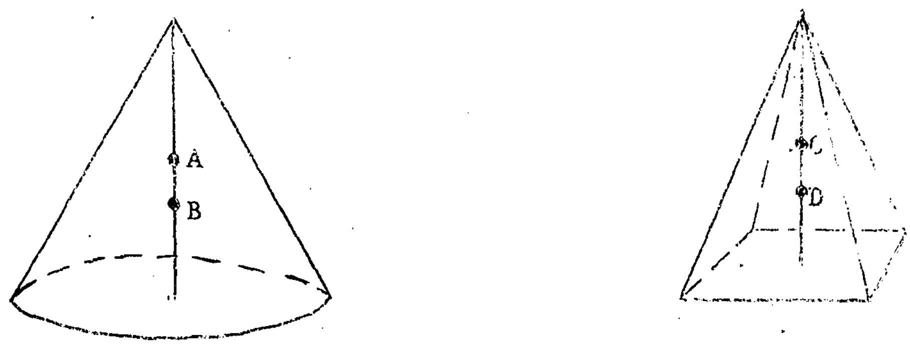


Figure 17

The geometric center of a cone or pyramid may be described as half-way between its apex and its base along the major axis. As you probably guessed, the center of gravity for each of these shapes, however, is not half-way along the major axis but rather close to the base than to the apex. No matter in what position you place a cone above a pointed support, it will be balanced if its center of gravity is immediately above the point of support along a vertical line.

Please go on to page 74.

In circular motion, we are most often concerned with spherical bodies. For our forthcoming work on satellites, it will suffice to consider satellites as spheres so that we can refer to them as bodies rather than particles. The radius of rotation is measured from the center of the circle of rotation to the center of gravity of the sphere.

The Earth possesses one natural satellite, the moon. The Space Age began with the dramatic ascent of a tiny object, Sputnik I, the first man-made satellite, which was successfully place into orbit by the U.S.S.R. Today literally hundreds of man-made moons revolve around the Earth in the outer fringes of our atmosphere.

Naturally, we are interested in the physics of a satellite's orbit. How fast must it move to stay in orbit? Must a heavier satellite move faster or more slowly than a lighter one to remain in the same stable orbit? How does the radius of the orbit (distance between the center of gravity of the satellite and the center of gravity of the Earth) affect the speed required to keep the satellite there? If we projected a satellite from Mars, would its speed be the same as for a similar orbit around the Earth?

We know enough to answer all these questions now! Let's take them one at a time. Before we do, however, we'll want to get a general qualitative picture of the satellite orbits.

Please go on to page 75.

Figure 18 shows a satellite being launched from point A. Its initial path is vertical or nearly so to point B. Automatically, or by radio control from the ground, the satellite vehicle is turned into path BC. At C the satellite separates from the last rocket stage and becomes a "free agent." Driving power, active from A through C, is now gone. The satellite's future behavior will be determined only by natural forces.

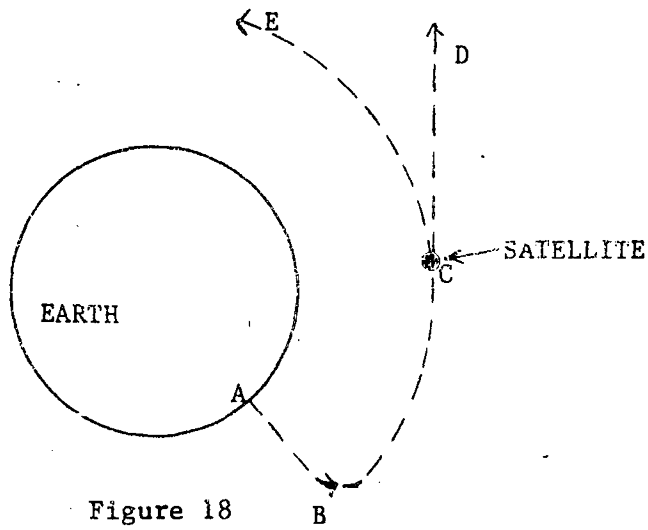


Figure 18

Two possible paths of the "free" satellite are shown in the diagram, CD and CE. Under what conditions would the satellite take path CD, a perfectly straight line?

(30)

A If a large amount of driving power had been supplied to it by its rocket at the moment before separation.

B If point B were at an altitude of at least 1,000 miles.

C If gravitational force from the Earth did not act on it.

YOUR ANSWER --- C

You are correct. The motional state that describes dynamic equilibrium is uniform velocity, a state in which neither the speed nor the direction of the motion changes.

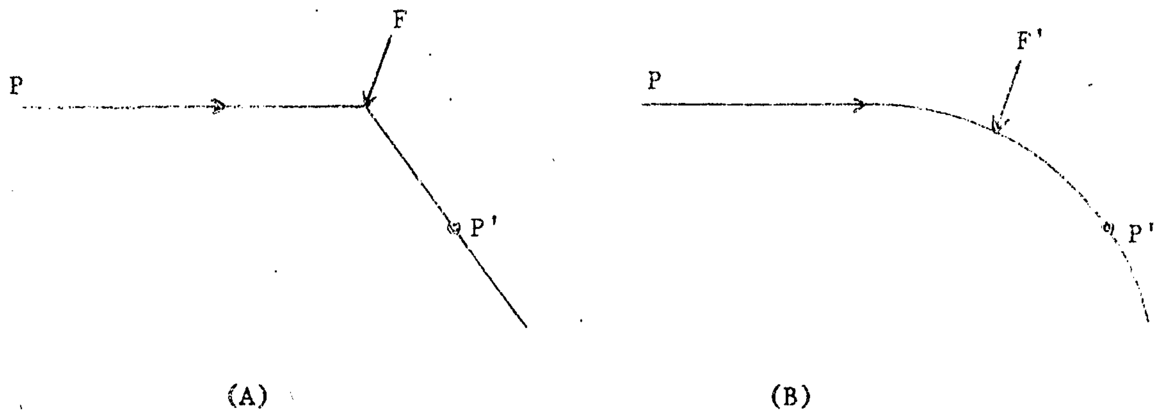


Figure 1

In Figure 1A, a particle P, initially moving with uniform velocity from left to right is acted upon by a downward force causing it to change its direction of motion as at P'. In Figure 1B, the same particle with the same initial velocity is acted upon again by a downward force, but you will notice at once that the appearance of this deflection is different than the previous one. We might describe the deflection in 1A as sharp while that in 1B might be described as gradual.

There are several possible explanations for the difference in deflection in the two cases. At this moment, however, we are concerned with just one of them. In the following list, only one of the possibilities could account for the difference in deflection. Which one is it?

(2)

A F is a pushing force, while F' is a pulling force.

B F' is acting for a longer time than F.

C F' is a force of larger magnitude than F.

D The particles in the two cases may have different masses and hence different inertias.

YOUR ANSWER --- C

Refer to Figure 13 on page 50.

Perhaps you did not notice that we said the string became slack again at the very instant the particle was restored to its position on the circle at point C. Thus, from C to D the string is slack and cannot exert a force on the particle. With no unbalanced force acting on it, the particle must have moved with uniform velocity from C to D. We must conclude, then, that it did not accelerate at all along this path.

Think it over. Then return to page 50 and choose a better answer.

YOUR ANSWER --- A

One of the conversions in this list is incorrect. It might help if we reminded you that there are 100 cm in one meter and 1,000 g in one kilogram.

When you locate the error, the list of values will be correct. Find it; then return to page 56 and select the right list.

YOUR ANSWER --- B

You are correct. The acceleration produced by an unbalanced force acting on a mass takes place in the direction of the force. ALWAYS! This is a physical law because there are no known exceptions to it.

Now we want to apply this idea to centripetal force.

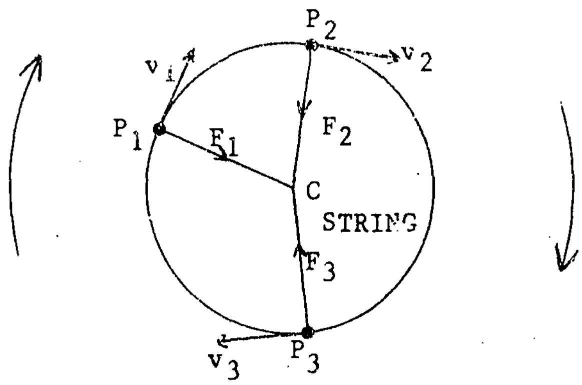


Figure 12

In Figure 12, a particle is moving in a circular path with uniform speed and is shown in three instantaneous positions, P_1, P_2, and P_3. The force required to maintain the circular motion for each instantaneous position is designated as F_1, F_2, and F_3. The force at each position is, of course, the centripetal force and, as shown, is directed toward the center of rotation. In accordance with the discussion we have just completed, if the force acts toward the center of rotation, then what must the particle do?

(25)

A It must accelerate away from the center of rotation at the instant of application of this force.

B It must accelerate toward the center of rotation at the instant this force is applied.

C It cannot be accelerating at all since its speed is constant.

YOUR ANSWER --- A

There are two errors in the combined form.

First, F_c was assumed to be directly proportional to v on the basis of rough experimental evidence. Your combined form shows an inverse proportion between these two variables.

Second, F_c was assumed to be inversely proportional to r, also on the basis of a rough experiment. Your combined form indicates a direct proportion between these variables.

Be sure to keep the inverse and direct proportion straight.

Please return to page 59. Study the remaining possibilities and make your next selection right.

YOUR ANSWER --- C

You are correct. The full solution follows:

$$F_c = \frac{mv^2}{r} = \frac{80 \text{ kg} \times (20 \text{ m/sec})^2}{20 \text{ m}}$$

$$F_c = \frac{80 \times 400}{20} \frac{\text{kg-m}}{\text{sec}^2} = 1{,}600 \text{ newtons}$$

Try another problem. This time, a 980-gram ball is whirled in a horizontal circle the radius of which is 36 cm. What is the maximum speed it can have if it is not to break the string? This particular string will break if 49 nt of force are exerted on it.

Note carefully that the units given are CGS rather than MKS, except for the breaking force. Before starting your solution, be sure to convert the units that need changing in order to get all the quantities in a single system of measurement.

Solve the problem and determine the maximum speed of the ball for the conditions described. The first step should be to solve the literal equation so that the unknown is alone on the left of the equals sign. Which of the following is the correct literal solution?

(18)

A $v = \sqrt{\frac{F_c r}{m}}$

B $v = \frac{F_c^2 r^2}{m^2}$

C $v = \frac{F_c r}{m}$

YOUR ANSWER --- B

This is not true. It may be that these actions and changes are so commonplace to you that it's difficult to stop and figure out what really happens. However, it is unreasonable to expect that the force will decrease with decreasing radius. Refer to Figure 10.

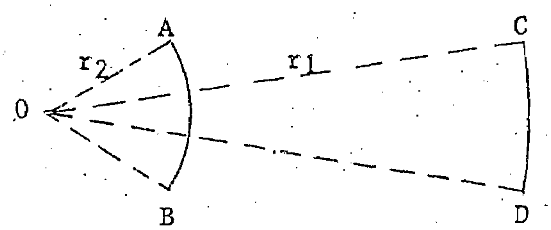


Figure 10

We have drawn two arcs, AB and CD, from the same center but with different radii. For comparison purposes the actual lengths of the arcs are very nearly the same. Note how flat CD is compared with AB. Or in other words, note how great the curvature of AB is as compared with CD.

Thus, arc CD approaches a straight line more closely than AB. This, in turn, means that CD is less of a departure from dynamic equilibrium than AB; hence less centripetal force would be required to produce circular motion when the radius is r_1 than when it is r_2.

Please return to page 30. Choose the other answer.

YOUR ANSWER --- D

Statement D is quite true but is not a summary of notebook entry 2(h). Check your notes again.

Please return to page 133 and select another possibility.

YOUR ANSWER --- B

No, that's not correct.

If $F_c = kv^3$, then the chart for these trials would look like this (taking k = 1):

	v	$kv^3 = F_c$
(Trial 1)	1.0 m/sec	$k(1)^3 = 1$ nt
(Trial 4)	2.0 m/sec	$k(2)^3 = 8$ nt
(Trial 5)	3.0 m/sec	$k(3)^3 = 27$ nt
(Trial 6)	4.0 m/sec	$k(4)^3 = 64$ nt

(Taking k = 1 is not actually necessary, but it does make our arithmetic somewhat simpler.)

Well, our chart does not show these experimental results for the trials listed. This means that the expression $F_c = kv^3$ does not meet the requirements of the data; hence it cannot be the correct relation.

Please return to page 42 and select a better answer.

YOUR ANSWER --- C

You are correct. The unbalanced force due to gravity acting on the satellite causes the satellite to follow a curved path (CE).

Like the planets and the moon, artificial satellites move in elliptical orbits. For our purposes, however, the assumption of a circular orbit to make our calculations easier will do no harm. So let us say that CE is the arc of a circle.

We are ready now to tackle the first question: How fast must the satellite move to stay in orbit?

The gravitational force between the satellite and the Earth is the centripetal force that causes the satellite to follow a circular orbit. From the Law of Universal Gravitation, we have:

$$F_g = G\frac{m_s m_e}{r^2}$$

This is the gravitational force that affects the motion of the satellite. The mass of the satellite is represented by m_s, the mass of the Earth by m_e, and G represents the constant of universal gravitation. (See notebook entry 1(d) for Lesson 9. You should refresh your memory in regard to the relative magnitude and units used for G.) The distance between the centers of gravity of the satellite and Earth is symbolized by r. What is the approximate value of r when the satellite is still on the Earth's surface?

Write your answer; then turn to page 86.

CORRECT ANSWER: At the Earth's surface, the value of r is approximately 4,000 miles.

We may take the separation between centers of gravity as the radius of the Earth, namely 4,000 miles. The satellite is so tiny compared to the Earth that its radius is inconsequential, even in precise calculations.

For the general situation of a satellite the centripetal force needed to keep it in orbit is gravitational force:

$$F_g = G\frac{m_s m_e}{r^2}$$

But we know that centripetal force is also given by the expression:

$$F_c = \frac{mv^2}{r}$$

In this expression for centripetal force, which mass does m symbolize for the motion of a satellite around the Earth?

(31)

A The mass of the satellite.

B The mass of the Earth.

C The combined mass of both Earth and satellite.

YOUR ANSWER --- B

Refer to Figure 3 on page 115. Between A and B, the particle moves in a straight line. Assuming that the speed is uniform as directed in the figure, then neither the speed nor the direction of the particle is changing. It is in dynamic equilibrium in this region and there are no unbalanced forces acting on it. The first part of your answer is, therefore, correct.

From E to F, however, the path is a smooth curve. As we have seen, a particle will traverse a curved path only if an unbalanced force acts on it to cause the direction of its velocity vector to change. There must be an unbalanced force acting on the particle from E to F. This part of your answer is therefore incorrect.

Please return to page 115. Choose a better answer.

YOUR ANSWER --- A

This doesn't even look right. And with units substituted, it comes out like this:

Given: $G = \frac{m^3}{kg\text{-}sec^2}$, $m_e^{\ 2} = kg^2$, $m_s = kg$, $r^3 = m^3$.

Substituting: $v = (\frac{m^3}{kg\text{-}sec^2})(\frac{kg^2 kg}{m^3})$

$$v = \frac{kg^2}{sec^2}$$

But velocity is not measured in the square of kilograms per second.

So, work out the problem carefully. Please return to page 20 and look over the other answers.

YOUR ANSWER --- A

This does not follow from a study of the equation you just derived.

A little earlier we showed that a proton must move more slowly in a given orbit than an electron in the same orbit with the same centripetal force applied because the proton is more massive. This probably helped you reach your conclusion.

The difficulty with this kind of reasoning is that you are prone to forget something. The magnetic force acting on the proton and electron was the same for both. But in connection with satellites, the heavier (or more massive) satellite will have more force acting on it since the force is due to gravitation. So the two situations are not the same.

Look at the equation critically. Does the mass of the satellite appear in it anywhere? What does this tell you about the dependence of the speed of the satellite on its mass?

Please return to page 52. Choose a better answer.

YOUR ANSWER --- B

There are three errors in the combined form you have selected.

First, F_c was assumed to be directly proportional to v on the basis of rough experimental evidence. Your combined form shows an inverse proportion between these two variables.

Second, F_c was assumed to be directly proportional to m, also on the basis of rough experiment. Your form shows an inverse proportion between F_c and m.

Third, F_c was assumed to be inversely proportional to r. Your choice indicates a direct proportion between these variables.

Somehow, there was an inversion of the position of all three variables. Be sure to keep the inverse and direct proportions straight.

Please return to page 59 and select an answer that meets the conditions of the problem.

YOUR ANSWER --- A

Refer to Figure 15 on page 46. If you placed a piece of chalk at Point B and then drew a circle around C using a piece of string of length r, would this circle coincide with the one already there? No, it would not. Yet, by this definition, r would be the radius of the new, smaller circle you just drew. So the radius of the original, larger circle extends more than from B to C.

Please return to page 46 and select another answer keeping the above discussion in mind.

YOUR ANSWER --- B

You're joshing us!

Make the stone massive enough, and it will pull you right off your feet as you try to whirl it through the air at the end of a string! A large mass has a large inertia and therefore tends to keep going in a straight line with a tremendous "determination." To overcome this inertia of motion for a large mass, what must be true of the magnitude of the centripetal force?

Please return to page 26. Choose the alternative answer.

YOUR ANSWER --- B

You are correct. The orbital radius is the sum of the Earth's radius and the altitude, or 4,000 mi + 250 mi = 4,250 mi.

Now look at the proportionality form of the speed equation:

$$v = \frac{k}{\sqrt{r}}$$

Our object is to determine the radius of a stable orbit for a new speed of 9,000 mi/hr. This is half the original speed. The next step may not be necessary for everyone, but it is straightforward and recommended.

Since the radius r is the unknown, we'll solve the proportion for r.

$$v = \frac{k}{\sqrt{r}} \quad \text{becomes} \quad \sqrt{r} = \frac{k}{v}$$

Squaring both sides, we get:

$$r = \frac{k}{v^2}$$

At half the original speed, we may write:

$$r' = \frac{k}{(\frac{1}{2}v)^2}$$ where r' is the new radius.

$$\text{or} \qquad r' = \frac{k}{\frac{1}{4}v^2} \qquad \text{or} \qquad r' = \frac{4k}{v^2}$$

Since $r = k/v^2$ and $r' = 4k/v^2$, then how many times as large is the new radius compared to the original radius?

Write your answer; then turn to page 94.

CORRECT ANSWER: The new radius must be 4 times as large as the original radius.

In short, if the speed is to be halved, the orbital radius will have to be quadrupled. The original radius was 4,250 miles, so the new radius will have to be 4 x 4,250 = 17,000 miles.

To find the altitude of the satellite, we subtract the radius of the Earth from the orbital radius and obtain 17,000 mi - 4,000 mi = 13,000 mi. Thus the new altitude is 52 times as great as the original altitude. A rather surprising answer, isn't it?

Summarizing our conclusions thus far:

(1) The speed of a satellite in orbit is given by the equation

$$v = \sqrt{G\frac{m_e}{r}}$$

(2) The mass of a satellite does not enter into the calculations of its speed.

(3) The speed of a satellite is inversely proportional to the square root of the orbital radius.

Our last question: If we projected a satellite from Mars, would its speed be the same as for a similar orbit around the Earth? By "similar orbit" we mean an orbit of the same radius, not the same altitude.

How would you answer this question?

(37)

A Yes.

B No.

C I don't know.

YOUR ANSWER --- D

Your answer is not correct; one of the equations given is a correct simplification of the original statement.

The original equation reads:

$$\frac{m_p v_p^2}{r_p} = \frac{m_e v_e^2}{r_e}$$

You are supposed to make use of the fact that the proton and electron move in circles that have the same radius. That is, $r_p = r_e$. Well, if $r_p = r_e$, then why not drop the subscripts and rewrite the equation this way:

$$\frac{m_p v_p^2}{r} = \frac{m_e v_e^2}{r}$$

A simplification is now possible with an r in the denominator of each fraction. Try multiplying both sides of the equation by r. What do you get?

Please return to page 17. You should have the answer now.

YOUR ANSWER --- B

You are correct. Refer to Figure 13 on page 50. The force F is applied over path BC, causing the particle to accelerate toward the center. We pointed out that the particle moved away from the center by the amount BC while it was moving from A to B; then it was accelerated toward the center along BC, exactly compensating for its retreat from the center over the previous path. This retreat and return would be repeated over and over as the string is first slackened and then tightened again. You might view this action as one in which the particle is repeatedly moving away from the center by the same amount that it is being accelerated toward the center, therefore keeping its average distance from the center the same over the circular path. So it is possible for a particle to accelerate toward the center of rotation without ever falling into it!

Please turn to page 97.

Now, suppose we made the interval between jerks smaller and smaller as in Figure 14. This would not alter the action as described above; it would simply cause the little central accelerations to occur more frequently so that the "sawtooth" motion would begin to look smoother and smoother. Finally, when the jerks were so closely spaced as to appear smooth, the path of the particle would become a ___________.

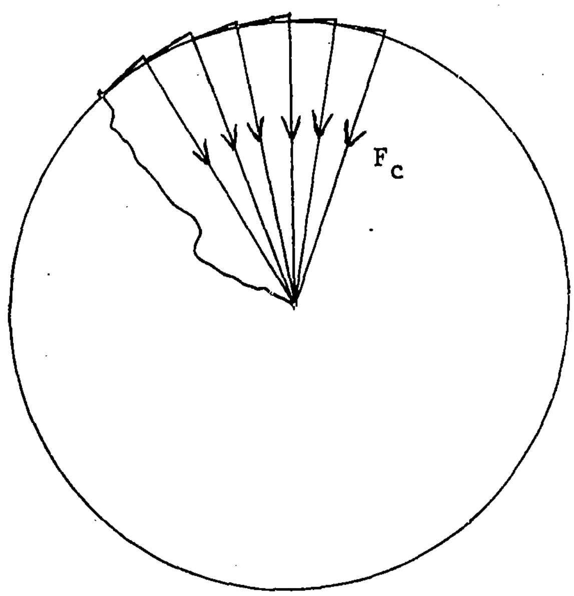


Figure 14

After completing the sentence above, please turn to page 98.

CORRECT ANSWER: When the jerks were so closely spaced as to appear smooth, the path of the particle would become a circle.

The foregoing analysis was intended to help you see that a particle moving in a circular path must accelerate toward the center continuously. If you wish, you can think of a circle as being composed of an infinite number of sawteeth, the leading edge of each tooth representing the "falling inward" of the particle to compensate for its outward motion during the other part of the tooth path.

NOTEBOOK ENTRY
Lesson 10

3. Centripetal acceleration

(a) When a particle moves in a circle, it accelerates continuously toward the center of rotation. This acceleration is a result of a change of direction of its velocity rather than a change of magnitude. The particle does not gradually approach the center of rotation because it is falling outward continuously due to inertia just as fast as it is falling inward due to the central acceleration.

Now, what is the magnitude of centripetal acceleration?

Like any other mass in motion, a particle moving in a circle must obey Newton's Second Law of Motion.

As a refresher, complete this statement: the general form of Newton's Second Law may be written as

(27)

A $a = \frac{F}{m}$

B $a = \frac{m}{F}$

C $a = Fm$

YOUR ANSWER --- A

All the known quantities have been expressed in the MKS system. You even went to some trouble to convert some of the data. This would mean that your answer could not be presented in the CGS system.

Without committing ourselves as to the accuracy of the numerical part of the answer above, we must insist on the result being given in MKS units. This is not meant as a trick, but we hope that you are learning to pay closer attention to the units as well as the numerical values of your answers.

Please return to page 31 and choose another answer.

YOUR ANSWER --- B

This won't do. You're doing some strange tricks with squares and square roots.

Let's do it one step at a time. The equation is:

$$F_c = \frac{mv^2}{r}$$

We want v, the unknown, on the left side all alone. Hence, we can multiply both sides by r/m to eliminate m and r from the right side.

$$F_c \times \frac{r}{m} = \frac{mv^2}{r} \times \frac{r}{m}$$

$$\frac{F_c r}{m} = v^2$$

$$v^2 = \frac{F_c r}{m}$$

We want v, not v^2, as the unknown. To do this we must take the square root of both sides. You evidently squared the right side rather than taking its square root. This was your error.

Please return to page 81. Repeat the manipulation; then choose the correct answer.

YOUR ANSWER --- A

Refer back to Figure 1 on page 76.

Of course, either of the two forces might be a pulling or a pushing force. No mention of this was made in the statement of the circumstances. However, whether the force pulls or pushes, its effect is quite the same. You may remember from your study of composition and resolution of forces in Lesson 7 that any force could be treated either way, and that there is no difference in result if we consider the force to be pulling or pushing.

Therefore, the answer you selected is not correct. You can't account for the difference in deflection on this basis.

Please return to page 76 and make another choice.

YOUR ANSWER --- B

You should be able to see the error of this answer by reviewing the situation from the point of view of inertia of motion. A proton is much more massive than an electron. This mass gives it more inertia. Therefore, a proton has a greater tendency to continue to move in a straight line than an electron. Now, to change its direction of motion--to make it move in a circle rather than along a straight line--you would have to exert a centripetal force on it. But if its tendency to keep moving in a straight line is stronger than that of an electron, how would this centripetal force compare with the force needed to duplicate the action for the electron?

Do you remember how force and mass are related in the expression $F_c = mv^2/r$?

Please return to page 140 and choose a better answer.

YOUR ANSWER --- C

You must have gotten the column headings mixed.

Look at Trials 4 through 6. Observe that the speed is not the same in these trials; it is 2.0 m/sec in the fourth trial, 3.0 m/sec in the next, and 4.0 m/sec in the next.

Observe, too, that in Trials 4 through 6 the radius is held constant, while your answer says that it was varied. It was varied in Trials 7 through 9.

Please return to page 118. Select an answer that fits the facts.

YOUR ANSWER --- B

You are correct. By assuming k to be dimensionless in the first place and then by substituting and discovering that F_c turns out to be in newtons as it should, we have justified the initial assumption.

A notebook entry is called for here.

NOTEBOOK ENTRY
Lesson 10

(Item 2)

(b) If a particle of mass m kg moves in a circle whose radius is r meters with a speed of v m/sec, then the centripetal force acting on it is:

$$F_c = \frac{mv^2}{r} \text{ newtons}$$

(c) Verbally, this equation may be given as: the centripetal force acting on a particle moving with uniform speed in a circle varies directly as the mass and the square of the speed and inversely as the radius of the circle of rotation.

Let's try a relatively easy problem involving these concepts.

An 80-kg man rides in a car which makes a sudden turn. He moves along a curve of radius 20 m at a speed of 20 m/sec. What centripetal force acts on him?

Write the equation, make the necessary substitutions including units, and then solve for the centripetal force. What is the correct answer?

(17)

A 80 nt

B 160 nt

C 1,600 nt

YOUR ANSWER --- C

The unknown is the speed of the satellite. In solving an equation, we always try to get the unknown alone on the left side of the expression, isolated from the other quantities.

In that case, you don't want to solve for the mass of the satellite.

Please return to page 111 and try again.

YOUR ANSWER --- C

No, this is not so. Refer back to Figure 1 on page 76. Without going into too much detail on the subject, isn't it fairly apparent that if F' had a greater magnitude than F, the deflection in B would tend to be sharper that the deflection in A? If a rubber ball is rolling along the ground past you, and you want to send it off on a new path by kicking it sideways, you would find that the harder you kick it at right angles to its path, the sharper would it curve off.

So, here is another factor upon which the sharpness of deflection depends: the magnitude of the force applied perpendicularly to the initial path. But forces of larger magnitude will cause sharper deflections, not the other way around.

Please return to page 76. Consider the remaining answers logically before making your next choice.

YOUR ANSWER --- B

If you note that m, the mass of the planet, appears in the numerator of the equation:

$$v = \sqrt{G\frac{m}{r}}$$

this should tell you at once that there is a direct kind of relationship between mass and speed. That is, as the mass increases, the speed increases; as the mass decreases, the speed decreases.

Thus, if the mass of Mars is only about 1/10 that of the Earth, then m for Mars would be 1/10 m_e. Would this make v for the Martian satellite larger or smaller than for the terrestrial satellite?

Please return to page 135; select the other answer.

YOUR ANSWER --- A

Your answer implies that the orbital speed is <u>independent of the mass</u> of the planet around which the satellite moves. In the equation:

$$v = \sqrt{G\frac{m_e}{r}}$$

we know that G is a universal constant, equally applicable to problems involving either Earth or Mars or any other planet; we also are working on the basis of the same orbit around Earth and Mars, so r is the same for each case; but how does m_e come into the picture? The subscript "e" was used to indicate that we were speaking of the mass of the Earth. For a Martian satellite, we would have to replace m_e with m_m, the mass of Mars.

From this, it should be clear that the orbital speed of the satellite is certainly dependent upon the mass of the planet around which it moves. The mass of Mars is about 1/10 that of the Earth.

Please return to page 94 and pick an alternative answer.

YOUR ANSWER --- A

This statement is not true! The particle is moving at uniform speed. All this means is that it traverses equal distances along the arc in equal time. But velocity is a vector quantity that can be fully described only by specifying both its magnitude and its direction. Two velocities may be said to be the same only if their magnitudes are equal and if they have the same direction.

A particle moving in a circle at any given instant in time has a velocity directed along the tangent to the circle at that point. We call this the instantaneous velocity. This concept will be more fully explained a bit later. Refer to Figure 8 on page 146. The tangent is, of course, perpendicular to the radius at that point, so the vector $\vec{v_1}$ is drawn at right angles to its radius (OP) and the vector $\vec{v_2}$ is similarly drawn perpendicular to its radius (OP'). Now, do the instantaneous velocities $\vec{v_1}$ and $\vec{v_2}$ have the same direction? Can they be said to be the "same?"

Please return to page 146. Select the correct answer.

YOUR ANSWER --- D

This is incomplete reasoning. The equation definitely gives the required information.

$$v = \sqrt{G\frac{m_e}{r}}$$

Here's a clue by analogy. In the study of falling bodies near the Earth's surface, we found that the final speed of a stone as it strikes the ground is given by the relation:

$$v = gt$$

if it started from rest. In this equation, g is the acceleration due to gravity, while t is the time of fall. Now we note that the mass of the stone does not appear in the equation. This means that the final speed of the falling body <u>does not depend on its mass at all</u> because, if a dependency existed between speed and mass, <u>then mass would show up in the equation</u>.

Now look at the equation above. Does the mass of the satellite appear in the equation? Do you remember that m_s canceled out during the simplification process? Now use your judgment.

Please return to page 52 and choose a better answer.

YOUR ANSWER --- A

You are correct. The mass of the Earth is not at all involved in this expression.

Thus far we have:

Centripetal force supplied by gravity:

$$F_g = G\frac{m_s m_e}{r^2}$$

and the general equation for centripetal force is:

$$F_c = \frac{m_s v^2}{r}$$

Note that we are using m_s rather than m in the second equation.

Since both expressions give the centripetal force on the satellite, then $F_g = F_c$, and we can equate the right sides of the equations. Thus,

$$\frac{m_s v^2}{r} = G\frac{m_s m_e}{r^2}$$ (Copy this on scrap paper.)

We are trying to answer the question: How fast must the satellite move to stay in orbit? To get the equation in the form most suitable for answering this directly, what should you solve it for?

(32)

A r

B v

C m_s

D I don't know.

YOUR ANSWER --- A

We have just shown that $F_c = kmv^2/r$ comes out in newtons when k is assumed to be unity and dimensionless and when the proper MKS units are substituted for the other quantities.

If, as you say, k is also measured in newtons, then we would have:

$$F_c = \underset{\text{(newtons)}}{k} \times \underset{\text{(newtons)}}{\frac{mv^2}{r}}$$

$$\text{or} \quad F_c = \text{(newtons)} \times \text{(newtons)}$$

$$F_c = \text{(newtons)}^2$$

So you see that taking k as measured in newtons forces us to come to an impossible conclusion. Force cannot be measured in (newtons)^2. Hence, k cannot be measured in newtons.

Please return to page 149 and make a better selection.

YOUR ANSWER --- A

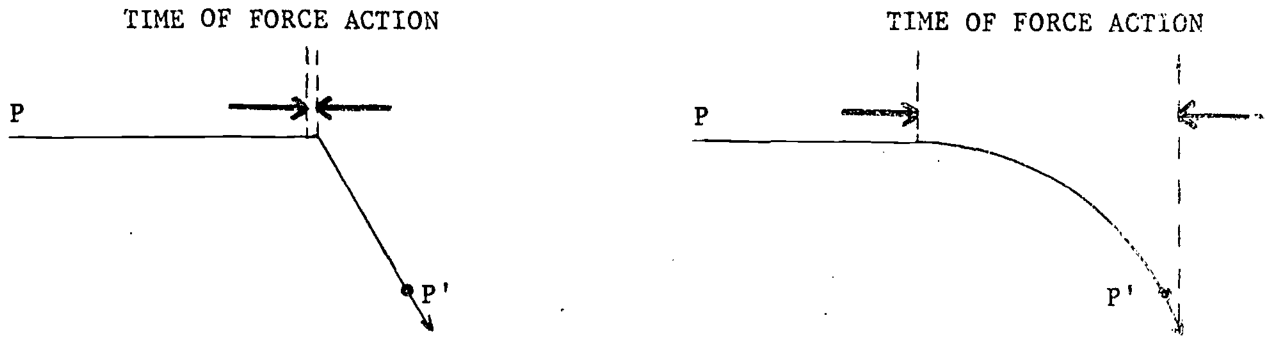
You have made a very common error. Your reasoning probably went something like this: since the centripetal force pulls inward toward the center of the circle, then when it is removed by cutting the string, the particle will move in the opposite direction or outward away from the center along the radius.

This is not so. You must remember that the particle is moving at a tangent to the circle at every instant in time and that the centripetal force is applied for the purpose of changing the motion from linear to circular motion. So, if the centripetal force causes the motion to change from linear to circular, the removal of the force must permit the particle to return to linear motion along the same line it was following at the instant when the string was cut. But was the particle flying outward from the center along the radius at the instant of cutting? Which way was it flying?

You should have no difficulty in choosing the correct answer now. Please return to page 150.

YOUR ANSWER --- B

You are correct. You can see that the curved part of the particle's path in B is longer than the curved path in A. Now let's see what this means.



(A) (B)

Figure 2

A moving particle will continue to change direction only as long as an unbalanced force acts on it. The moment the force is removed, the particle is restored to dynamic equilibrium and returns to its uniform velocity motion. Since a curve in a path signifies steadily changing direction, then the unbalanced force must be acting on the particle throughout the time that it is moving in the curved path. Figure 2 shows the relative durations of the forces for the two cases.

Please go on to page 115.

To check your grasp of this idea, suppose you now refer to Figure 3.

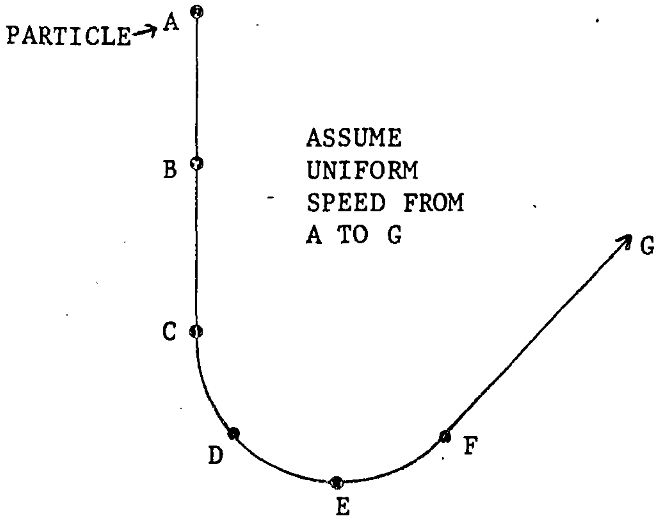


Figure 3

Over which ranges of the particle's path is there no unbalanced force acting on it? (If you wish to see the information on the previous page once more before answering the question, turn to page 114.)

(3)

A A to C and D to E.

B A to B and E to F.

C A to C and F to G.

D Only over A to C.

YOUR ANSWER --- C

When you say this, you imply that the centripetal force needed to produce a certain circular path at a certain speed is completely independent of the mass of the moving particle. Surely you can't mean that! The dependence is evident both from your own experience in whirling light and heavy objects on strings and from the mathematical analysis we have just completed. The following expression definitely states that the force required is directly proportional to the mass of the moving particle.

$$F_c = \frac{mv^2}{r}$$

The force needed to keep a proton in the same circle at the same speed as an electron is very different from the force applied to the electron.

Please return to page 140 and select a better answer.

YOUR ANSWER --- B

You are correct in your selection of this answer.

The fact that we do not get force units on the right side of the expression above shows that one (or more) of the following must be true:

F_c may not be directly proportional to m,
or F_c may not be directly proportional to v,
or F_c may not be inversely proportional to r,
or k may not be unity or it may have units of its own or both.

Short of a rigorous derivation of the equation for the magnitude of centripetal force in terms of the mass and speed of the rotating particle and the radius of the circle it describes, there is only one other way to find out which of the above statements is or are actual fact. We must perform a quantitative experiment in which we determine the actual effect of m, v, and r on the centripetal force.

Figure 11 is a chart showing the results of one possible experiment along these lines. Copy this chart into your notebook.

DETERMINING THE RELATIONSHIP OF F_c TO m, v, AND r.

	MASS (m) KG	SPEED (v) M/SEC	RADIUS (r) M	MEASURED FORCE IN NEWTONS (F_c)
TRIAL 1	1.0	1.0	1.0	1.0
TRIAL 2	2.0	1.0	1.0	2.0
TRIAL 3	3.0	1.0	1.0	3.0
TRIAL 4	1.0	2.0	1.0	4.0
TRIAL 5	1.0	3.0	1.0	9.0
TRIAL 6	1.0	4.0	1.0	16.0
TRIAL 7	1.0	1.0	2.0	0.50
TRIAL 8	1.0	1.0	3.0	0.33
TRIAL 9	1.0	1.0	4.0	0.25

Figure 11

Please go on to page 118.

Study the numerical results of the "experiment" carefully. You will see almost at once that the experiment was performed by allowing only one variable at a time to change value while observations were made of the effect of this variation on the centripetal force, F_c. All units used are preferred MKS units. The numbers are given to two significant figures, except in one case where we violated the rules just a bit to retain clarity.

As a random example, look at the figures on page 117 for Trial 7. The mass of the particle was 1.0 kg; its linear speed in the circle was 1.0 m/sec; the radius of the circle of rotation was 2.0 m. Then when the centripetal force needed to maintain this circular motion was measured, it turned out to be 0.50 nt.

As a further check on your interpretation of the chart of Figure 11, suppose you pick out the only true statement below.

(14)

A In Trials 1 through 3 mass and radius were held constant, while the speed was made to vary.

B In Trials 4 through 6 mass and radius were held constant, while the speed was made to vary.

C In Trials 4 through 9 mass and speed were held constant, while the radius was varied.

D In Trials 7 through 9 speed and radius were held constant, while the mass was varied.

YOUR ANSWER --- C

Possibly you are not sufficiently familiar with the characteristics of a parabola to answer this question. Without going into unnecessary detial on this subject, we can show you quite easily why curve DH cannot be part of a parabola.

Refer again to Figure 4 on page 43. Your attention was directed to the fact that the string's length was constant during the interval when the particle was describing curve DH. If you tried to draw a parabola (see Figure 6) with a pencil on one end of a string while the remote end of the string was connected to a fixed point, you would find it impossible to do so. A parabola is the kind of curve that opens outward as it is drawn to greater and greater lengths; its two sides never rejoin each other. A small portion of a parabola resembles a small portion of an equivalent ellipse. The difference is that the ellipse is a closed figure while the parabola is not.

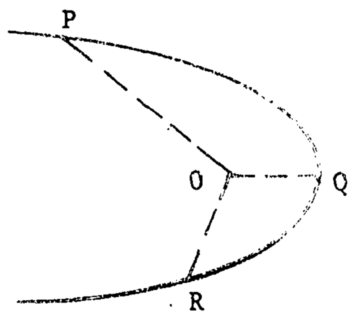


Figure 6

In Figure 6, you can see that there is no point inside a parabola that can be described as a center. If you select a point such as O, it is impossible to draw a parabola using fixed lengths of string as radii from this point. We will explain the focus later.

Please return to page 44 and try again.

YOUR ANSWER --- C

You are correct! Good work! You observed that the mass (m_s) of the satellite does not appear in the equation; hence the orbital speed (v) is completely independent of the satellite's mass. This means that a large, heavy satellite must orbit the Earth at the same speed as a small, light one if both are to have the same orbit.

Is this result surprising? In some ways it is because one might have the feeling that a massive satellite should move more slowly in a particular orbit than a large one. This intuitive thinking obviously has a flaw in it; we tend to forget that increasing the mass of a satellite does two things which cancel each other: (1) it increases the required centripetal force but (2) it also increases the gravitational pull of the Earth on the satellite, thereby providing the extra centripetal force.

All right! Are you ready to work on a practical problem involving satellite speed in a predetermined orbit? We hope so.

We want to orbit a satellite at 400 km above the Earth. What speed in meters per second will it need to stay in this orbit? Let's work to three significant figures. The data you will need to take down are:

$G = 0.667 \times 10^{-10}\ m^3/kg\text{-}sec^2$

$m_e = 5.98 \times 10^{24}$ kg

Altitude of satellite = 400 km

Radius of Earth = 6.37×10^6 meters

Now turn to page 121.

We must first establish the value of r, the orbital radius. Since 400 km = 4.00 x 10^5 m, we will add this figure to the radius of the Earth to obtain the radius of the satellite's orbit.

To add figures in scientific notation, we must be sure the exponents are the same, so we'll convert 4.00 x 10^5 to 0.400 x 10^6 and then add:

$$\begin{array}{r} 6.37 \times 10^6 \text{ m} \\ + \ 0.40 \times 10^6 \text{ m} \\ \hline r = 6.77 \times 10^6 \text{ m} \end{array}$$

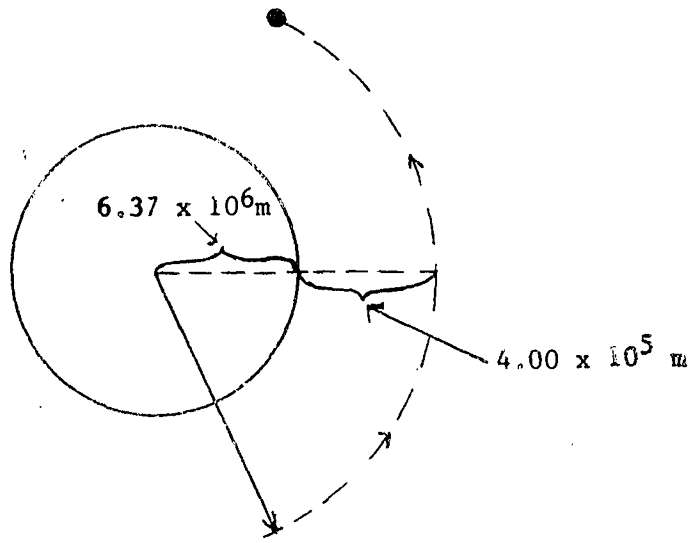


Figure 19

We now have what we need to substitute in the speed equation:

$$v = \sqrt{G\frac{m_e}{r}} = \sqrt{\frac{0.667 \times 10^{-10} \times 5.98 \times 10^{24}}{6.77 \times 10^6}}$$

How about a little exercise in arithmetic? Work it out and get the value of v in meters per second. Write your answer.

Please turn to page 122 for an answer check.

CORRECT ANSWER: To three significant figures, the satellite's velocity is <u>$v \approx 7,670$ meters per second</u>.

This turns out to be about 17,200 miles per hour. You may recall that our orbiting astronauts traveled at just about this speed as long as they remained in orbit, nearly 250 miles above the Earth's surface.

Looking back at what we've accomplished thus far, we have answered the first two of our self-imposed questions. First, the speed required of a satellite to keep it in orbit is given by the following equation:

$$v = \sqrt{G\frac{m_e}{r}}$$

Second, we have found that the mass of the satellite does not affect the speed calculations.

Our next question was: <u>How does the radius of the orbit affect the speed required to keep the satellite there?</u> We can get the answer to this directly from an inspection of the equation.

The required speed varies inversely as what?

(35)

A Radius of the orbit.

B As the square of the radius.

C As the square root of the radius.

YOUR ANSWER --- C

You're right! Both answers are wrong.

Since you were able to recognize the errors in the answers given, we expect that you probably have the right one.

CORRECT ANSWER: The maximum speed of the ball is 4.2 meters per second.

The solution: $v = \sqrt{\frac{F_c r}{m}} = \sqrt{\frac{49 \text{ nt} \times 0.36 \text{ m}}{0.98 \text{ kg}}}$

$$v = \sqrt{18} \text{ m/sec} = \underline{4.2} \ \underline{\text{m/sec}}$$

If F_c, r, and m are all in MKS units, it almost goes without saying that the velocity must come out in meters per second. But if you're interested in proving it, look at this:

Since a newton is a kilogram-meter per second^2, then:

$$\sqrt{\frac{\text{nt} \times \text{m}}{\text{kg}}} = \sqrt{\frac{\frac{\text{kg-m}}{\text{sec}^2} \times \text{m}}{\text{kg}}}$$

But kg in the denominator cancels kg in the numerator, and m x m = m^2, so we come out with:

$$\sqrt{\frac{\text{m}^2}{\text{sec}^2}}$$

Since this fraction is under the radical the final unit for v is m/sec.

Please go on to page 124.

Our final illustrative problem in the applications of centripetal force will deal with the motions of subatomic particles under special conditions. In certain types of particle accelerators or "atom smashers," as well as in other practical devices, charged particles such as electrons, protons, alpha particles, and so on are whirled around in circular paths by electric and magnetic forces. In particular, magnetic force is often used to exert the required centripetal effect, that is, to keep the particles moving in a circle rather than flying off at a tangent. Magnetic forces are like any other forces; they can change the speed or direction of motion of a moving mass. If you wish, you can picture a magnetic force as behaving like the string that constrained the ball or the stone to a circular path.

In what direction must the magnetic force act?

(21)

A Along a tangent to the circle in which the particle moves.

B At right angles to the tangent of the circle in which the particle moves.

YOUR ANSWER --- D

You must have gotten the column headings mixed.

In Trials 7 through 9 the mass and speed are kept constant, while the radius is varied. The latter is taken first at 2.0 m, then at 3.0 m, and finally at 4.0 m. So you have chosen an incorrect answer.

Please return to page 118. Then please make another selection.

YOUR ANSWER --- D

You didn't read the specifications of the problem with sufficient care. In Figure 1B on page 76, the same particle is acted upon by the downward force. If the particles are the same in both cases, how could the mass change from one case to the next?

You can avoid this type of error by exercising care in reading and interpreting each question.

Please return to page 76 and make a new selection.

YOUR ANSWER --- A

The unknown is the speed of the satellite. Don't you always try to get the unknown on the left side of the equation, alone and isolated from the other terms? Of course you do.

Then, please don't solve for the radius of the orbit.

Return to page 111 and choose a better answer.

YOUR ANSWER --- C

This is not correct. The fact that k is unity is not shown by the substitution of units in the proportionality above. The only way to determine the numerical value of a constant like k is to do an experiment in which all the values are measured. For example, in the chart given, it is evident that k equals 1 since nowhere in the list of figures would any other value yield the correct result for F_c for the given values of m, v, and r.

Refer to Figure 11 on page 117.

Choose one of the trials at random, say, Trial 6. Only if k = 1 can you obtain a force of 16.0 nt for a mass of 1 kg, a speed of 4 m/sec, and a radius of 1.0 m:

$$F_c = k\frac{mv^2}{r} = 1 \times \frac{1\text{ kg} \times 16\text{ m}^2}{1\text{ m}} = 16.0\text{ nt}$$

Please return to page 149 and choose another answer.

This page has been inserted to maintain continuity of text. It is not intended to convey lesson information.

YOUR ANSWER --- B

You are correct. Despite the uniform speed of the particle, its direction is constantly changing; hence the velocity $\vec{v}_1$ is different from the velocity $\vec{v}_2$.

Refer to Figure 8 on page 146.

Just what kind of velocities are we dealing with here? We speak of $\vec{v}_1$ as the <u>instantaneous</u> velocity of the particle at point P; likewise, $\vec{v}_2$ is the <u>instantaneous</u> velocity of the particle at point P'. A time instant has no time duration; and if no time is allowed for the particle to move, it can't cover any distance at all. Thus, it is difficult to visualize the nature of the velocity of a particle that doesn't move! It would serve you well at this time to think of an instant as beeing of <u>extremely short</u> duration (but not zero time) of say one-millionth, or one-billionth, or even one-trillionth of a second. Regardless of the brevity of the interval, the particle will move a minute distance. The moment we can "see" it in motion, even though the distance it travels may be ridiculously small, our picture of instantaneous velocity becomes real and actual.

Picturing the instantaneous velocity this way, we then draw the vector to represent it along the __________ to the circle at the point in question.

Write the missing word. Then turn to page 131.

CORRECT ANSWER: We draw the vector representing the instantaneous velocity along the tangent to the circle at the point in question.

Refer again to Figure 8 on page 146. Over a brief instant in time, the direction of motion of a particle in a circular path is considered to be along the tangent to the circle at that point. So in Figure 8 on page 146, $\vec{V}_1$ is the tangent to the circle where radius OP intersects it; similarly, $\vec{V}_2$ is the tangent at the point of intersection of radius OP'.

NOTEBOOK ENTRY
Lesson 10

1. Instantaneous Velocity

(a) Instantaneous velocity is defined as the velocity of a particle during an infinitesimal time interval, or a time interval so short as to be considered negligible.

(b) The direction of the instantaneous velocity of a particle moving in a circular path is that of the tangent to the circle at the point in question, in the direction of motion.

(c) Since the tangent is perpendicular to the radius at the point in question, then the direction of the instantaneous velocity is perpendicular to that radius. (Note: Copy Figure 8 on page 146.)

Returning to the physics of the boy and the particle he deflects, the force exerted on the particle by his hand must be directed inward along the particular radius the string happens to form at that instant. In Figure 8, OP and OP' are two such radii; hence $\vec{F}_1$ symbolizes the force at one instant and $\vec{F}_2$ symbolizes the force a short time later. What is the angular relationship between each force and the corresponding velocity vector?

(6)

A 0 degrees.

B 90 degrees.

YOUR ANSWER --- D

You may never have studied the properties of ellipses, parabolas, and other curves of this family, but you certainly have worked with circles.

We'll work on the assumption that you know nothing about any curved figure except the circle.

How do you define a circle? There are several definitions that can be quoted, but we'll consider only this one: a circle is a geometric figure having a center such that this center is equidistant from all points that lie on the circle.

All right? Now look at curve DH in Figure 4 on page 43. The particles may be considered to be points that "lie on the curve." Are all of the particle positions equidistant from point O? If they are, then DH is the arc of a circle; if they are not, then DH may be part of an ellipse, parabola, hyperbola, cycloid, cardioid, or what have you.

Please return to page 44. The right answer is almost self-evident.

YOUR ANSWER --- A

You are correct. The proton has more inertia and hence a stronger inclination to maintain straight-line motion. A greater force is needed to make it duplicate the electron's motion.

Now let's return to the original electron-proton problem. It should be somewhat clearer to you why a proton acted upon by the same force as an electron and moving in the same circle must move more slowly. If it moves with less speed, then it is possible for the same force to produce the same radius of rotation.

Before continuing, please turn to page 157 in the blue appendix.

NOTEBOOK CHECK

Refer to notebook entry 2(h) under Newton's Laws of Motion (Lesson 8). Which of the following is the best summary of this notebook item?

(24)

A The acceleration produced by an unbalanced force is smaller if the mass of the body being accelerated is larger.

B The acceleration produced by an unbalanced force acting on a mass takes place in the direction of the force.

C If the mass is measured in kilograms and the force in newtons, then the resulting acceleration will be in meters per second per second.

D Acceleration does not occur merely because forces act on a mass. A second condition is that the forces must be unbalanced.

YOUR ANSWER --- B

You are correct.

Using all four assumptions, we then went ahead to substitute units for the quantities on the right side of the expression above, omitting k from consideration since we had assumed it to be unity and dimensionless. After simplifying, we found that the units of mv/r came out kg/sec. A newton, however, is a kg-m/sec^2, so we see that the relation will not give us newtons.

We are then forced to the conclusion that one or more of the following statements are true:

Perhaps F_c is not directly proportional to m.
Perhaps F_c is not directly proportional to v.
Perhaps F_c is not inversely proportional to r.
Perhaps k does not equal unity.
Perhaps k has units of its own.

Each one of these invalidates one of the assumptions we made.

Now return to the original question and choose the correct answer. Turn to page 66.

YOUR ANSWER --- B

You are correct. The value of m_e in the speed equation would be different. Since v is a function of the mass of the planet around which the satellite orbits, then v would be different for a Martian satellite in an orbit of the same radius as that of a similar Earth satellite.

Here are the comparative masses of the two planets:

Earth: 6.0×10^{24} kg
Mars: 6.4×10^{23} kg

Very roughly, the Earth has about 10 times the mass of Mars. Using the equation:

$$v = \sqrt{G\frac{m}{r}}$$

(where m = the mass of the particular planet about which we want to orbit a satellite), let's answer this question: Would a Martian satellite travel more slowly or more rapidly than an Earth satellite having the same orbital radius?

(38)

A More slowly.

B More rapidly.

YOUR ANSWER --- A

You overlooked the radical. Perhaps if you rewrite the equation this way:

$$v = \sqrt{G}\,\frac{\sqrt{m_e}}{\sqrt{r}}$$

(a perfectly legitimate form), you can see that the speed is not inversely proportional to the radius of the orbit. The radius appears under the radical on the right side of the expression.

Please return to page 122 and choose a better answer.

YOUR ANSWER --- B

Judging from this answer, we would say that you did all of the arithmetic properly but omitted the last step called for by the equation. After multiplying and dividing all the factors under the radical, what must you do? Remember, you are trying to find v, not v^2.

Please return to page 31. You should be able to get the right answer now.

YOUR ANSWER --- B

You forgot the radical. The speed v is directly proportional to the <u>square root</u> of the planet's mass.

Please return to page 40 and select a more reasonable answer.

YOUR ANSWER --- B

You are correct. Since the radii are equal, they may be eliminated by multiplying both sides of the original equation by r.

You have been asked to find the speed that the proton would need to have to rotate in a circle of the same radius as the electron when the same magnetic force acts on it. The unknown in the above equation is, therefore, v_p. First solving for v_p^2 and then taking the square root of both sides, we have:

$$v_p = v_e\sqrt{\frac{m_e}{m_p}}$$

All the quantities at the right are known:

$$v_e = 3.0 \times 10^6 \text{ m/sec}$$
$$m_e = 9.1 \times 10^{-31} \text{ kg}$$
$$m_p = 1.6 \times 10^{-27} \text{ kg}$$

So the solution is now a purely mechanical matter. Solve for v_p in meters per second to two significant figures and write your answer before turning to page 140.

CORRECT ANSWER: $v_p = 7.2 \times 10^4$ m/sec.

The substitutions and numerical solution follow:

$$v_p = v_e\sqrt{\frac{m_e}{m_p}} = 3.0 \times 10^6\sqrt{\frac{9.1 \times 10^{-31}}{1.6 \times 10^{-27}}}$$

$$v_p = 3.0 \times 10^6 \sqrt{5.7 \times 10^{-4}}$$

$$v_p = 3.0 \times 10^6 \times 2.4 \times 10^{-2} = \underline{7.2 \times 10^4 \text{ m/sec}}$$

The result shows that the speed of the proton will be about 1/40th that of the electron.

We want you to be very clear in your own mind regarding the inevitability of this result. The proton is a massive particle compared to the electron--about 1,840 times as massive.

If you wanted a proton to move in a circle of the same radius as that of the electron's path and at the same speed, the centripetal force applied by the magnetic field would have to be:

(23)

A Larger than that applied to the electron.

B Smaller than that applied to the electron.

C The same as that applied to the electron.

YOUR ANSWER --- A

Well, let's see. First let's write the equation:

$$F_c = \frac{mv^2}{r}$$

Now let's substitute:

$$F_c = \frac{80\ \text{kg} \times (20\ \text{m/sec})^2}{20\ \text{m}}$$

So F_c is obtained by squaring the speed, multiplying by the mass, and then dividing by the radius. In order to get an answer of 80 nt as you did, you must have forgotten to square the speed. Because you see if you don't square 20 m/sec, you can cancel this number against the 20 m in the denominator, leaving 80 as the numerical result.

Please return to page 104. Go through the calculation again and determine the correct answer.

YOUR ANSWER --- A

This is not an assumption!

We're committed to the MKS system by mutual agreement. Force is measured in newtons in the MKS system. Therefore, we should set up every relationship in which force is involved so that the force does come out in newtons.

For the sake of consistency and mutual understanding between all workers in physics, we must never allow ourselves to be forced into a situation where our basic quantities cannot be measured in the units that everyone has agreed to use. This is a self-imposed "must." Thus, we must measure mass in kilograms, speed in meters per second, radii in meters, force in newtons, and so forth.

Whenever we develop a new physical equation, we do what is necessary to (1) phrase the equation to fit the experimental evidence. That is, the equation must work in all real-life situations in which it appears. Then, (2) we must adjust the constant of proportionality so that all the variables may be expressed in the chosen units of measure.

You should remember that we did exactly this in obtaining the final form of the gravitational force equation:

$$F_g = G\frac{m_1 m_2}{r^2}$$

We were forced to make $G = 6.7 \times 10^{-11}\ m^3/kg\text{-}sec^2$ in order to be able to express m in kilograms, r in meters, and F_g in newtons.

Please return to page 15. Make the other selection.

YOUR ANSWER --- A

No, you're allowing your terms to get mixed. Refer to Figure 8 on page 146. Consider $\vec{F_1}$. The direction of the $\vec{F_1}$ vector is inward along the radius OP. This force must be directed along the radius, as we emphasized previously, because reaction to it keeps the string taut. Thus the line of action of $\vec{F_1}$ is along OP.

The corresponding velocity vector at the instant when $\vec{F_1}$ is applied is $\vec{v_1}$. But we have shown that $\vec{v_1}$ lies along the tangent at the point where this vector touches the circle.

Then at the instant when $\vec{F_1}$ exists as a real force, it cannot be applied to the velocity vector at 0 degrees, can it? What is the angular relationship of a radius to a tangent at a given point on any circle?

Please return to page 131. The alternative answer is clearly the correct one.

YOUR ANSWER --- D

We are attempting to solve the equation in such form as to obtain the speed of the satellite in orbit. The speed of the satellite is designated by v in this expression:

$$\frac{m_s v^2}{r} = G\frac{m_s m_e}{r^2}$$

Since we want to find v, then the usual procedure should be followed: manipulate the terms algebraically, doing whatever is required to shift them legitimately, canceling where possible, until you wind up with v all alone on the left side of the equation. There are various ways to do this; we'll show you the method we prefer in a moment.

In any event, if you want v alone on the left side, the equation should be solved for this term.

To continue, please turn to page 20.

YOUR ANSWER --- A

Look again!

In Trials 1 through 3 the mass was varied, not held constant, while the other two quantities were not changed. Note that the particle is given a mass of 1.0 kg in Trial 1, 2.0 kg in Trial 2, and 3.0 kg in Trial 3. Note also that the speed was not made to vary at all, being held at 1.0 m/sec throughout these three trials.

Please return to page 118. Then pick an alternative answer.

YOUR ANSWER --- A

You are correct. Figure 7 below is a magnified reproduction of curve DH of Figure 4 on page 43. The string is held at constant length; hence it serves as a radius of the arc described by rotating the radius around fixed point O. The force applied by the boy's hand on the particle as it moves along arc DH is directed inward toward the center at all times so that the particle is forced to change its direction in a smooth, continuous manner.

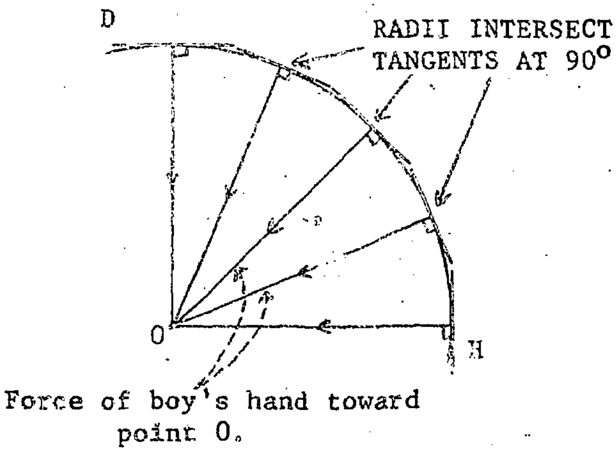


Figure 7

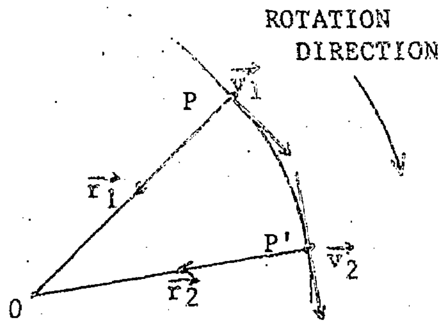


Figure 8

We have added tangents to the circular arc to show how the radii intersect the circle. As you must know from plane geometry, a radius is <u>always</u> perpendicular to a tangent at the common point of intersection. Now let us use this fact to crystallize a very important concept relating the <u>direction</u> of the boy's force to the <u>direction</u> of the particle motion <u>at any given instant</u>.

In Figure 8 above, particle P is shown in two different positions as it moves in the arc produced by the force of the boy's hand on the string. It is moving with <u>uniform speed</u>. Does this mean that its velocity at the instant shown by v_1 is the same as its velocity at the instant shown by v_2?

(5)

A Yes.

B No.

YOUR ANSWER --- A

You are absolutely correct. Testing this is easy:

	v	$kv^2 = F_c$ (taking $k = 1$)
(Trial 1)	1.0 m/sec	$k(1)^2 = 1.0$ nt
(Trial 4)	2.0 m/sec	$k(2)^2 = 4.0$ nt
(Trial 5)	3.0 m/sec	$k(3)^2 = 9.0$ nt
(Trial 6)	4.0 m/sec	$k(4)^2 = 16.0$ nt

The values in the second column are identical with those given in the chart; hence $F_c = kv^2$ is correct. That is, the centripetal force varies as the square of the speed of the particle in circular motion.

Our original assumption that the centripetal force is directly proportional to the speed thus proves to have been incorrect.

As the last step, we must check the third assumption we made, namely, that $F_c = k/r$, or that the centripetal force is inversely proportional to the radius of rotation. Assuming $k = 1$, perform steps similar to those above. You will find in Trials 1, 7, 8, and 9 that: as r is doubled from 1.0 m to 2.0 m, the value of F_c is reduced to one-half; as r is tripled from 1.0 m to 3.0 m, the value of F_c is reduced to one-third; and as r is quadrupled, the centripetal force is reduced to one-fourth its original magnitude.

Do the data for these trials indicate an inverse proportion between F_c and r? Write your answer; then turn to page 148.

CORRECT ANSWER: Yes, the data for Trials 1, 7, 8, and 9 indicate that F_c is inversely proportional to the radius of rotation.

Thus, $F_c = \frac{k}{r}$.

Do you remember that we made a fourth assumption, namely, that k = 1? The experimental results in Trial 1 alone demonstrate that this assumption was justified because if k were any number except unity, we could not get a force of 1 nt when a mass of 1 kg is whirled at 1 m/sec in a circle of 1 m radius. Whether or not k is dimensionless will be proved very shortly.

Before we do this, we want to write the new, correct form of the combined proportion. Three of the four original assumptions are the same as they were, but one of them has changed:

$F_c = km$	We have shown this to be justified.
$F_c = kv^2$	The initial assumption ($F_c = kv$) has been shown to be wrong.
$F_c = k/r$	We have shown this to be justified.
$k = 1$	We have shown this to be justified.

Now we want you to combine these proportionalities just as you did before, taking into account the change in the second one.

Write your answer. Then turn to page 149.

CORRECT ANSWER: The three separate proportions may be combined in the form:

$$F_c = k\frac{mv^2}{r} \text{ where } k = 1.$$

We do not yet know, however, whether k is dimensionless. We shall proceed to find out.

To do this, we'll substitute MKS units on the right side of the equation and simplify.

$$F_c = \frac{mv^2}{r} = \frac{(kg) \times (m/sec)^2}{m}$$

$$F_c = \frac{kg \times \frac{m^2}{sec^2}}{m}$$

$$F_c = \frac{kg\text{-}m}{sec^2} = \underline{\text{newtons}}$$

You will have observed that we did not introduce k or units for it in the above substitution. Despite this, we find that the relation $F_c = mv^2/r$ does turn out to have newtons as the unit of measure.

What does this tell you about the constant of proportionality, k?

(16)

A k is measured in newtons.

B k is dimensionless.

C k has a numerical value of unity.

YOUR ANSWER --- B

You are correct. At any instant the force causing the circular motion lies along the radius while the corresponding velocity vector is tangent to the circle. A radius is always perpendicular to a tangent at a given point on the circle.

So we must conclude that any particle in motion will follow a curved path only if an inward radial force is applied to it, this force being instantaneously perpendicular to the direction of the particle's motion at that instant.

This inward radial force is called centripetal (sen trip et al) force. It is the force applied by the center of rotation on the rotating body. (We will discuss "centrifugal force" later in this lesson.)

Referring once again to Figure 8 on page 146, imagine that someone cuts the string when the particle is at position P'. At the instant of cutting, the centripetal force F_2 ceases to exist, since it is only through the medium of a continuous string that the center can apply the required force to the particle. Then, at the instant of cutting and thereafter, how will the particle move if nothing disturbs it?

(7)

A Outward, along the radius, away from the center.

B Inward, along the radius, toward the center.

C At a tangent to the circle, along the line of $\vec{v_2}$.

YOUR ANSWER --- C

Well, let's look at the speed equation again:

$$v = \sqrt{G\frac{m_e}{r}}$$

We want to know if v will be the same for a Martian satellite traveling in an orbit of the same radius (r) as an Earth satellite. Looking at the terms on the right, we know that:

(1) G is a universal constant and hence is the same on Mars as on Earth or anywhere else in the universe.

(2) r is the same for the Martian and Earth satellites. This is given in the conditions of the problem.

(3) m_e is the mass of the Earth. This is the mass we use in calculating the orbit of an Earth satellite. To calculate the orbit of a Martian satellite, we would use the mass of Mars; hence the value of m will be different for the two calculations. The mass of Mars is about 1/10 that of the Earth.

If one of the terms on the right side of the equation is different for two problem solutions, can the dependent variable on the left be the same in both cases?

Please return to page 94 and choose the right answer.

YOUR ANSWER --- B

You are correct. You remembered that in any situation involving action and reaction (Newton's Third Law), there are two bodies upon which the forces act. Call the string Body A; call the ball Body B; then Body A exerts a centripetal force on Body B, so Body B reacts by exerting a centrifugal force on Body A.

The hand that holds one end of the string while the ball is being whirled around on the other end is actually the site of the centripetal force in the first place. But this force is transmitted through the string to the ball; it is more convenient to think of the string as exerting the inward force on the ball, while the ball exerts the outward force on the string.

So, we have redefined centrifugal force. No longer is it a phony, fictional force erroneously conceived as being applied to the ball. Now it is a real, outward-acting force applied to the string, owing its existence to centripetal force and developing as a reaction to the inward force.

Before continuing, please turn to page 159 in the blue appendix.

You have now completed the study portion of Lesson 10 and your Study Guide Computer Card and A V Computer Card should be properly punched in accordance with your performance in this Lesson.

You should now proceed to complete your homework reading and problem assignment. The problem solutions must be clearly written out on 8½" x 11" ruled, white paper, and then submitted with your name, date, and identification number. Your instructor will grade your problem work in terms of an objective preselected scale on a Problem Evaluation Computer Card and add this result to your computer profile.

You are eligible for the Post Test for this Lesson only after your homework problem solutions have been submitted. You may then request the Post Test which is to be answered on a Post Test Computer Card.

Upon completion of the Post Test, you may prepare for the next Lesson by requesting the appropriate

1. study guide
2. program control matrix
3. set of computer cards for the lesson
4. audio tape

If films or other visual aids are needed for this lesson, you will be so informed when you reach the point where they are required. Requisition these aids as you reach them.

Good Luck!

WORKSHEET

Please listen to Tape Segment 1 of Lesson 10 before starting to answer the questions below. Use Computer Card for answers.

QUESTIONS

1. As you know, the earth revolves about the sun in its yearly perambulations. If you wanted to describe the forces acting in this system, which one of the following would you choose as your point of observation in order to establish an outside frame of reference?

   A The Earth.
   B The Sun.
   C A nearby star.
   D A nearby comet
   E The Moon

2. Just before the Apollo 8 astronauts took off for the Moon, their spacecraft circled the Earth in a temporary orbit. To analyze the forces acting on the spacecraft while orbiting,

   A the Sun would have been a satisfactory point of observation because it is motionless in space.
   B The Moon would have been a satisfactory point of observation because an observatory could be set up there.
   C The Sun would be satisfactory because it is "fixed" relative to both the Earth and the spacecraft.
   D the planet Mars would be satisfactory because it is "fixed" relative to the Earth.
   E None of these answers is correct.

3. One can build up a set of consistent physical laws based on "centrifugal" force provided that one is

   A inside the framework of the rotating system.
   B outside the framework of the rotating system.
   C conscious that centrifugal force acts on the rotating particle or body.
   D willing to assign algebraic signs to vectors.
   E None of these is correct.

Please return now to page 5 of

the STUDY GUIDE

WORKSHEET

Please listen to Tape Segment 2 of Lesson 10 before starting to answer the questions below. Use the Computer Card for answers.

QUESTIONS

4. A plumb line passing through a horizontal plane makes an angle of

   A 180° with the plane.
   B 90° with the plane.
   C Zero degrees with the plane.
   D 90° with the vertical.
   E None of these answers is correct.

5. Aside from the centripetal force of the string, another force that acts on the whirling particle is gravitation. Which statement is true?

   A Gravitation acts on the particle only if it is moving in a horizontal plane.
   B Gravitation acts on the particle if it is moving in any plane.
   C Gravitation acts on the particle only if it is moving in a vertical plane.
   D Regardless of the plane in which it moves, the effect of gravitation on the centripetal force required to keep the particle in an "orbit" is always the same.
   E None of these answers is correct.

6. A horizontal circle is best for the circumstances described in the STUDY GUIDE because

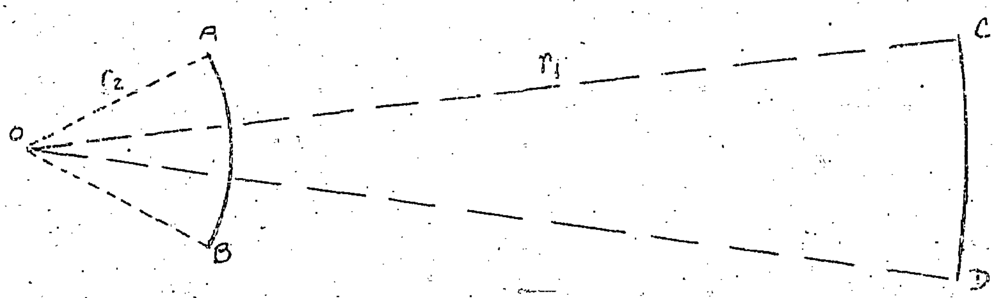
   A it is the easiest to observe as you whirl the particle around your head.
   B in this kind of circle there will be less danger of the particle striking some nearby object.
   C in any other plane, the particle may move in an elliptical rather than a circular path.
   D only in this kind of circle can the centripetal force be equal for all instantaneous positions of the particle.
   E None of these answers is correct.

Please return now to page 10 of THE STUDY GUIDE.

WORKSHEET

Please listen to Tape Segment 3 of Lesson 10 before starting to answer the questions below. Use Computer Card for answers.

Data Item A: The drawing below is a duplicate of Figure 10 in the STUDY GUIDE for this Lesson.



Data Item B: The planets of the Solar System, in random order, are: Earth, Pluto, Saturn, Mars, Mercury, Venus, Neptune, Jupiter, and Uranus.

QUESTIONS

7. For a particle moving with uniform speed in a circle, which one of the following statements is true?

   A Centripetal force on the particle is proportional to the tangential velocity, all other quantities equal.
   B Centrifugal force acting on the rotating particle is directly proportional to the mass of the particle.
   C Centripetal force is independent of the mass of the rotating particle.
   D Centripetal force is independent of the radius of the circle described by the particle.
   E Centripetal force is inversely proportional to the radius of the circle described by the particle.

8. If orbital radius were the only factor governing the magnitude of centripetal force, then which one of the planets would be acted on by the largest force and which the smallest?

   A Largest: Pluto; smallest: Mercury.
   B Largest: Uranus; smallest: Jupiter.
   C Largest: Earth; smallest: Venus.
   D Largest: Mercury; smallest: Pluto.
   E None of the above is correct.

Please return to page 26 of the STUDY GUIDE

WORKSHEET

Please listen to Tape Segment 4 before starting to answer the questions below. Use the Computer Card for answers.

Data Item A: Centripetal force = $F = \frac{mv^2}{r}$

Data Item B: Mass of α-particle = 4 times mass of proton and Centripetal force acting on α-particle moving in same magnetic field as proton = 2 times force on proton.

QUESTIONS

9. If the orbital radius of a proton in a given magnetic field is *r*, what orbital radius would an α-particle assume if it moved into the same field at an identical speed?

   A r/4

   B r/3

   C r/2

   D 4r

   E 2r

Please return now to page 133
in the STUDY GUIDE.

WORKSHEET

Please listen to Tape Segment 5 of Lesson 10 before starting to answer the questions below. Use Computer Card for answers.

Data Item A: Study the chart below.

	A	B	C	D
(1) Mercury	2.4×10^8	4.79×10^6	1.47×10^{19}	3.5×10^5
(2) Venus	6.1×10^8	3.50×10^6	3.29×10^{20}	1.0×10^6
(5) Jupiter	7.1×10^9	1.31×10^6	1.27×10^{23}	5.97×10^6
(7) Uranus	2.5×10^9	0.68×10^6	5.81×10^{21}	2.10×10^6

One of the columns above lists the approximate orbital velocities of the planets shown.

QUESTIONS

10. Which column lists the approximate orbital velocities of the planets shown?

A A

B B

C C

D D

11. Jupiter is by far the most massive planet in the Solar System. Which of the columns above lists the masses of the planets shown?

A A

B B

C C

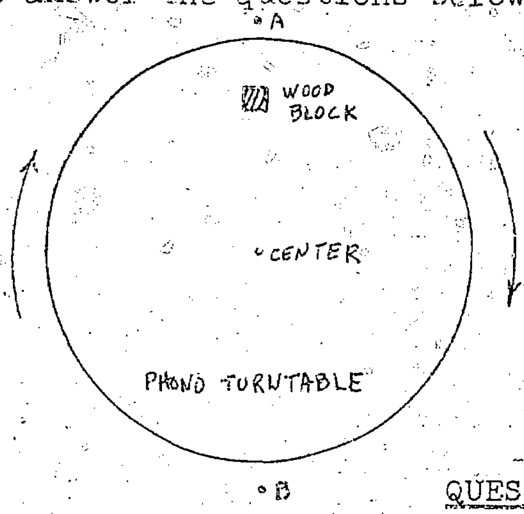
D D

E None of the columns lists the masses of the planets.

Please return now to page 64 of the STUDY GUIDE

WORKSHEET

Please listen to Tape Segment 6 of Lesson 10 before starting to answer the questions below. Use Computer Card for answers.



The diagram at the left shows a small wood block resting on the surface of a slowly rotating phono turntable. Shown above the turntable is a reference point labeled "A"; below it is another reference point labeled "B". The turntable is rotating clockwise as seen from above.

QUESTIONS

12. At the instant shown in the diagram, on what does the centrifugal force act and toward which of the two reference points?

    A On the turntable, toward point A.
    B On the turntable, toward point B.
    C On the wood block, toward point A.
    D On the wood block, toward point B.
    E None of these is correct.

13. Suppose the turntable were rotating counterclockwise instead of clockwise. Which one of the following would be affected by this change of direction?

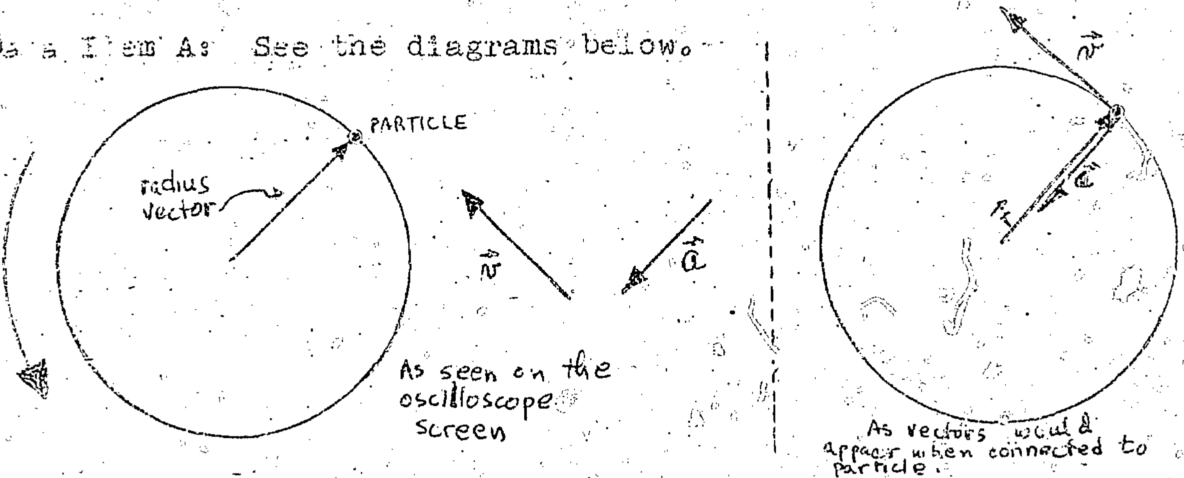
    A The wood block.
    B The surface of the turntable.
    C The centripetal force.
    D The centrifugal force.
    E None of these.

Please go directly to page 160 of the STUDY GUIDE after answering the questions above.

WORKSHEET

Please listen to Tape Segment 7 of Lesson 10. You will also need to view the brief film entitled THE VELOCITY AND ACCELERATION IN CIRCULAR MOTION as directed on the audio tape.

Data Item A: See the diagrams below.



Data Item B: Select (by letter) one of the following as an answer to each question below:

A tangent; B radius; C parallel; D perpendicular

QUESTIONS

14. When a particle moves in a circle, its instantaneous velocity vector always has the same direction as the ______ to the circle at that point.

15. At every instant, the centripetal acceleration vector is ______ to the velocity vector.

16. At every instant, the centripetal acceleration vector lies along a ______ of the circle of rotation.

Please go to page 153 of the STUDY GUIDE.

AMP LESSON 10

HOMEWORK PROBLEMS

1. What is the centripetal acceleration of a car which moves around a circular section of a road having a radius of 150 ft at a constant speed of 75 ft/sec?

2. A ball on a string is whirled in a horizontal circle of radius 2 ft. What must be the speed of the ball if its centripetal acceleration is to be equal to g? (Take g = 32 ft/sec/sec).

3. What centripetal force is needed to keep a 2-kg mass moving at a constant speed of 4 m/sec in a circle having a radius of 4 m?

4. A force of 10 nt applied to one end of a cord keeps an object tied to the other end moving at a speed of 2 m/sec in a horizontal circle of 8 m radius. What is the mass of the object?

5. At a point 6.7×10^6 m from the center of the Earth, g is approximately 9.0 m/sec/sec. What velocity must be given to an Earth satellite to send it into orbit at this distance? (Assume a circular orbit).

6. A student swings a mass of 100 grams tied to one end of a cord in a horizontal circle of radius 50 cm so that its tangential velocity is 100 cm/sec. Calculate the centripetal force acting on the mass in newtons.

7. What is the centripetal force acting on each kiligram of an airplane that is turning at 200 m/sec in a horizontal circle of radius 10,000 m?

8. Using the figures given below, estimate the centripetal force acting on the Earth as a result of its rotation around the Sun:

   Mass of Earth = 6.0×10^{24} kg

   Radius of Earth orbit = 1.5×10^{11} m

   Time required for Earth to complete one orbit = 3.2×10^7 sec

   (Hint: For an object moving in a circle, $v = \frac{2\pi r}{T}$ in which v = tangential velocity, r = radius of rotation, and T = time for one revolution.)